1.

$$\lim_{x\to\infty}\frac{\left(2-\frac{1}{x}\right)\left(\frac{3}{x}-1\right)}{\left(1-\frac{1}{x}\right)\left(1+\frac{3}{x}\right)}=\frac{2\bullet -1}{1\bullet 1}=-2$$

Answer B

2.

Answer D

$$\int x^{-2}dx=-\frac{1}{x}+c$$

3.

$$f\,'=(x-1)(3)\left(x^2+2\right)^2(2x)+\left(x^2+2\right)^3\bullet 1$$

$$f\,'=\left(x^2+2\right)^2\left((3x-3)(2x)+x^2+2\right)$$

Answer D

$$f\,'=\left(x^2+2\right)^2\left(6x^2-6x+x^2+2\right)$$

$$f\,'=\left(x^2+2\right)^2\left(7x^2-6x+2\right)$$

4.

$$\frac{1}{2}\int\sin(2x)2dx+\frac{1}{2}\int\cos(2x)2dx$$

Answer B

$$=-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+c$$

Answer A

5.

$$\lim_{x\to 0}\frac{x^2\left(5x^2+8\right)}{x^2\left(3x^2-16\right)}=\lim_{x\to 0}\frac{\left(5x^2+8\right)}{\left(3x^2-16\right)}=-\frac{1}{2}$$

6.

$$f=\begin{cases}x+2\text{ when } x\neq 2\\ 1\text{ when } x=2\end{cases}$$

This function has a limit at x = 2 . The limit is equal to 4. But this is not the function value at x = 2 so the function is not continuous. If the function is not continuous at x = 2, the function is not differentiable at x = 2.

Answer A

7. $$2+\int_0^1 v(t)dt=2+\left(t^3+3t^2\Big|_0^1\right)=2+(1+3)-0=6$$

Answer B

8.

$$f'=3\sin(3x)$$

$$f'\left(\frac{\pi}{9}\right)=3\sin(3\bullet\frac{\pi}{9})=-\frac{3\sqrt{3}}{2}$$

Answer E

9.

From the graph we know that for -2<x<0 g'=f is positive and for 0<x<1 g'=f is also positive so in these intervals g is increasing. For 1<x<2 g'=f is negative so g is decreasing in this interval. Therefore the function g reaches a maximum when the function g changes from increasing to decreasing. This happens at x = 1.

Answer D

10.

The right Riemann sum will be the least value because all the rectangles will be inscribed under the graph of f.

Answer C

The left will be the greatest value. The midpoint will be between the left and right. The trapezoidal will be greater than the right Riemann sum.

11.

Studying the graph of f we can see that the function has positive slope, then negative slope, and then positive slope. There is a local maximum to the left of zero and a local minimum to the right of zero.

Answer B

The graph that illustrates this is graph B

12.

Answer D

$$f'(x) = -\frac{2}{x^2}e^{\frac{2}{x}}$$

13.

Using the chain rule:

Answer A

$$f'(\ln x)\cdot\frac{1}{x} = \frac{2\ln x + 2}{x}$$

14. For a change in concavity to take place there must be a change in the sign of f". The chart tells us only the sign of f" at four values of x. From the chart we can see that in the interval from 0<x<2 the function does change sign.

Answer E

At first glance you might think there is a local maximum at x=1, but there is not sufficient information to make this conclusion since we don't know about the value of f at points in a small neighborhood around x = 1.

At another glance you might think that x = 1 is a point of inflection, but the definition of an inflection point is that it must change sign. Again we don't know what is happening in a small neighborhood around x = 1.

15. Using u substitution:

$$\frac{1}{2}\int \frac{2x}{x^2-4}\,dx$$

$$u = x^2 - 4$$

$$du = 2x\,dx$$

Answer C

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c$$

$$\frac{1}{2}\ln\left|x^2-4\right| + c$$

16. Using implicit differentiation:

$$\cos(xy)\cdot\left(x\frac{dy}{dx}+y\right)=1$$

$$x\frac{dy}{dx}+y=\frac{1}{\cos(xy)}$$

$$x\frac{dy}{dx}=\frac{1}{\cos(xy)}-y$$

Answer D

$$x\frac{dy}{dx}=\frac{1-y\cos(xy)}{\cos(xy)}$$

$$\frac{dy}{dx}=\frac{1-y\cos(xy)}{x\cos(xy)}$$

17.

We first know that g"=f ' and from the graph we know that for 0<X<2 and 5<x<6 f ' is positive and for 2<x<5, f ' is negative. Therefore, g" changes sign at x = 2 and x = 5 only. These are the locations for the points of inflection.

Answer C

18. The tangent line is y = k-x. This means that slope at some point is -1. We know that $y' = 2x + 3$ so

$$-1 = 2x + 3$$

$$x = -2$$

Therefore,

Answer A

$$y(-2) = 4 - 6 + 1 = -1$$

so

-1=2+k or k = -3

19.

$$\lim_{x \to -\infty} \frac{5 + 2^x}{1 - x} = 5$$

$$\lim_{x \to \infty} \frac{\frac{5}{2^x} + 1}{\frac{1}{2^x - 1}} = \frac{1}{-1} = -1$$

Answer E

This question was not scored.

20.

From the given second derivative we know that for $-\infty<x<0$, $0<x<.3$, and $6<x<\infty$ f" is positive . We also know that for $3<x<6$, f" is negative, therefore f" changes sign at x = 3 and x = 6 so these are the locations of the points of inflection.

Answer D

21.

v' would be indicated by studying the slope of x(t). For $0<t<2$, v' is positive and for $3<t<6$ v' is negative. Therefore, we know that the velocity is increasing when v' is positive. $(0<t<2)$

Answer A

22. Reading the statement

Rumor spreads at a rate: $\frac{dP}{dt}$

number of people who heard the rumor: p

number of people that have not heard the rumor: N-p

Product of number of people who heard the rumor and number of people who have not heard the rumor: (p)(N-p)

Answer B

k= constant of proportionality

Therefore:

$$\frac{dP}{dt} = k(p)(N-p)$$

23.

$$y\,dy = x^2\,dx$$
$$\int y\,dy = \int x^2\,dx$$
$$\frac{y^2}{2} = \frac{x^3}{3} + C$$
$$y^2 = \frac{2x^3}{3} + K$$
$$y = -\sqrt{\frac{2x^3}{3} + K}$$

Answer E

$$-2 = -\sqrt{\frac{2(3)^3}{3} + K}$$
$$4 = 18 + K$$
$$k = -14$$
$$y = -\sqrt{\frac{2x^3}{3} - 14}$$

24.

Tangent line: $y = 4(x-2)+1$

Approximate value along tangent line:

Answer B

$$y(1.9) = 4(1.9-2)+1$$
$$= -0.4 + 1$$
$$= 0.6$$

25.

$$f' = \begin{cases} c & x \le 2 \\ 2x - c & x > 2 \end{cases}$$

$$\lim_{x\to 2^+} f' = 4 - c$$

$$\lim_{x\to 2^-} f' = c$$

$$\therefore c = 4 - c$$

$$c = 2$$

f must also be continuous so **Answer B**

$$\lim_{x\to 2^+} f = \lim_{x\to 2^-} f$$

$$2\bullet 2 + d = 4 - 4$$

$$d = -4$$

so c+d = -2

26.

$$4x = \tan y$$

$$4 = \sec^2 y \bullet \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4}{\sec^2 y}$$

Answer A

$$\frac{dy}{dx} = \frac{4}{\sec^2\left(\frac{\pi}{4}\right)} = \frac{4}{\left(\sqrt{2}\right)^2} = 2$$

27.

Studying the slope field we notice that at x = -1 the slope of the function represented by the slopefield has a slope of zero. This is true for all values of y. Therefore (x+1) must be a factor of the derivative of the function.

When x >-1 and y>0, the slope of the function represented by the slope field is positive and increases as x and y increase.

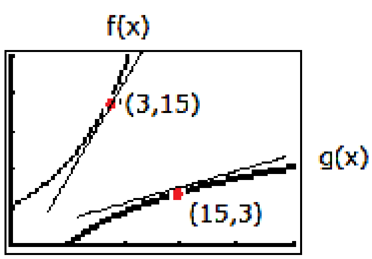
When x<-1 and y>0, the slope of the function gets represented by the slope field is negative and gets steeper as x decreases and y increases.

At y=0 the slope of the function represented by the slope field is zero, therefore y must be a factor of the derivative of y.

Answer C

Therefore, $\frac{dy}{dx} = xy + y = y(x+1)$

28.



Answer A

Using the diagram above that represent f and g being inverse functions, we know that the slope of f at (3,15) is the reciprocal of the slope g at (15,3).

We know that f ' (3) = -2, so g ' (15) = -1/2.