The Calculus AB Exam

CALCULUS AB

A CALCULATOR **CANNOT BE USED ON PART A OF SECTION I.** A GRAPHING CALCULATOR FROM THE APPROVED LIST **IS REQUIRED FOR PART B OF SECTION I AND FOR PART A OF SECTION II** OF THE EXAMINATION. CALCULATOR MEMORIES NEED NOT BE CLEARED. COMPUTERS, NONGRAPHING SCIENTIFIC CALCULATORS, CALCULATORS WITH QWERTY KEYBOARDS, AND ELECTRONIC WRITING PADS ARE NOT ALLOWED. CALCULATORS MAY NOT BE SHARED AND COMMUNICATION BETWEEN CALCULATORS IS PROHIBITED DURING THE EXAMINATION. ATTEMPTS TO REMOVE TEST MATERIALS FROM THE ROOM BY ANY METHOD WILL RESULT IN THE INVALIDATION OF TEST SCORES.

SECTION I

Time— 1 hour and 45 minutes

All questions are given equal weight.

Percent of total grade—50

Part A: 55 minutes, 28 multiple-choice questions
A calculator is NOT allowed.

Part B: 50 minutes, 17 multiple-choice questions
A graphing calculator is required.

Parts A and B of Section I are in this examination booklet; Parts A and B of Section II, which consist of longer problems, are in a separate, sealed package.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE YOUR ANSWERS TO QUESTIONS IN PART A ON PAGE 2 OF THE SEPARATE ANSWER SHEET. THE ANSWERS TO QUESTIONS IN PART B SHOULD BE INDICATED ON PAGE 3 OF THE ANSWER SHEET. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example:

What is the arithmetic mean of the numbers 1, 3, and 6 ?

(A) 1

(B) $\frac{7}{3}$

(C) 3

(D) $\frac{10}{3}$

(E) $\frac{7}{2}$

Sample Answer

Ⓐ Ⓑ Ⓒ ● Ⓔ

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $y = \left(x^3 + 1\right)^2$, then $\frac{dy}{dx} =$

(A) $\left(3x^2\right)^2$ (B) $2\left(x^3 + 1\right)$ (C) $2\left(3x^2 + 1\right)$ (D) $3x^2\left(x^3 + 1\right)$ (E) $6x^2\left(x^3 + 1\right)$

CHAIN RULE! (derivative of the outside; leave the inside alone. Then multiply by the derivative of the inside.)

$2(x^3+1) * 3x^2 = 6x^2(x^3+1)$

2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$

$\int e^{kx} = \frac{e^{kx}}{k} + C$

$\left.\frac{e^{-4x}}{-4}\right|_0^1 = \frac{e^{-4}}{-4} - \frac{e^0}{-4} = \frac{e^{-4}}{-4} + \frac{1}{4}$

2003 Released Exam Excerpt

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

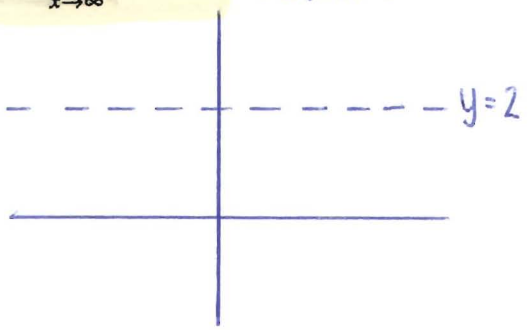
(A) $f(0) = 2$ False because $f(0)$ is undefined

(B) $f(x) \neq 2$ for all $x \geq 0$ False because $x \leq 0$ also

(C) $f(2)$ is undefined. False. We dont know a function.

(D) $\lim_{x \to 2} f(x) = \infty$ False. $\lim_{x \to 2} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = 2$ Definition of limits.



4. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

QUOTIENT RULE!

u = numerator

V = denominator

$$\frac{vu' - uv'}{v^2}$$

$u = 2x+3 \quad u' = 2$

$V = 3x+2 \quad v' = 3$

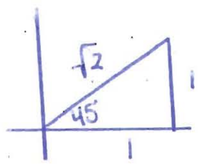
$$\frac{dy}{dx} = \frac{(3x+2)2 - (2x+3)3}{(3x+2)^2}$$

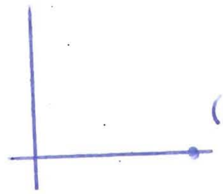
$$= \frac{6x+4 - (6x+9)}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$$

5. $\int_0^{\frac{\pi}{4}} \sin x \, dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

$-\cos x \Big|_0^{\pi/4} = \underbrace{-(\cos \pi/4)}_{45°} + \cos 0 = -\frac{1}{\sqrt{2}} + 1$ (rationalize denominator)





(1,0)

$= \frac{-\sqrt{2}}{2} + 1$

6. $\lim\limits_{x \to \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

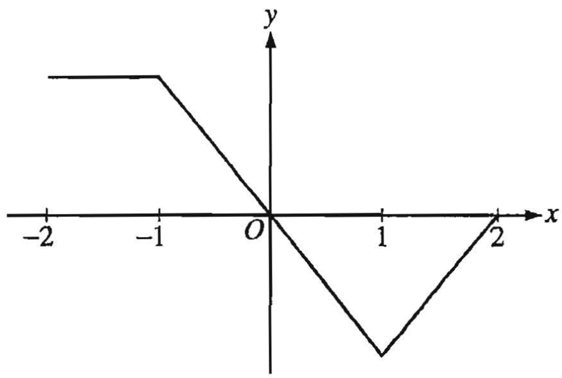
(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) −1

$\frac{x^3}{4x^3} = \frac{1}{4}$

* degree of numerator < degree of denominator $\Rightarrow \lim\limits_{x \to \infty} = 0$

* " " " > " " " $\Rightarrow \lim\limits_{x \to \infty} = \text{DNE}$

* " " " = " " " $\Rightarrow \lim\limits_{x \to \infty} = \frac{a}{b}$



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$. False. $(0 \le x \le 2)$ below x-axis

(B) f is increasing for $-2 \le x \le 0$. True. Above x-axis.

(C) f is increasing for $1 \le x \le 2$. False. above x-axis

(D) f has a local minimum at $x = 0$. False. minimum $\Rightarrow$ from below to above.

(E) f is not differentiable at $x = -1$ and $x = 1$. False. This is velocity graph. Not position.

8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

NO PRODUCT RULE!
↳ U-SUBSTITUTION

$u = x^3$
$du = 3x^2\, dx$
$\frac{du}{3} = x^2 dx$

$\int \cos u * \frac{du}{3} = \frac{1}{3}\int \cos u\, du$
$= \frac{1}{3}[\sin u + c]$
$= \frac{1}{3}[\sin(x^3) + c]$

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

CHAIN RULE!

$\frac{d}{dx}(\ln u) = \frac{1}{u}\, du$

$\frac{d}{dx}(e^u) = e^u du$

$u = x + 4 + e^{-3x}$
$du = 1 - 3e^{-3x}$
$\frac{dy}{dx} = \frac{1}{x+4+e^{-3x}} \cdot (1 - 3e^{-3x})$

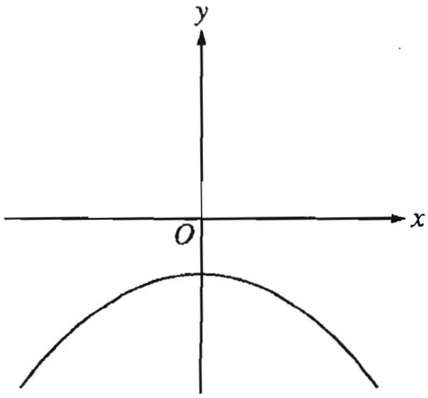
$f'(0) = \frac{1 - 3e^0}{4 + e^0} = \frac{-2}{5}$

2003 Released Exam Excerpt

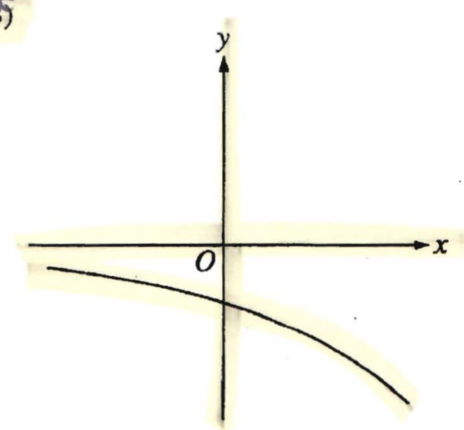
10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

negative concave down
slope

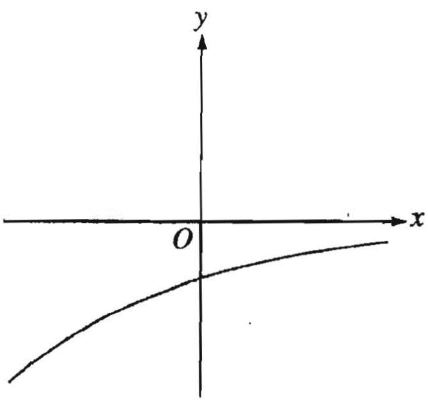
(A)



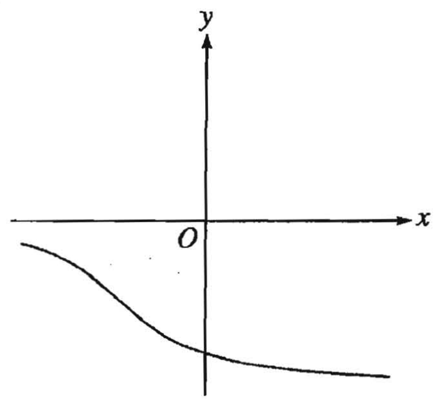
(B)



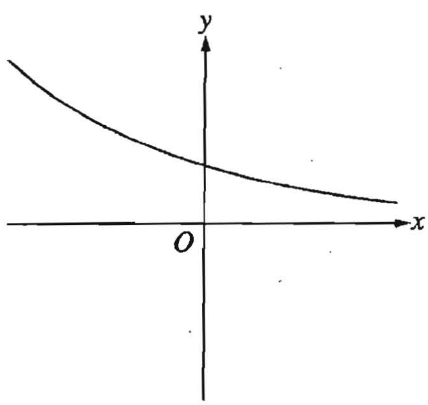
(C)



(D)



(E)



11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

* change limits of integration

$u = 2x + 1$
$du = 2dx$
$\frac{du}{2} = dx$

$x = 0 \Rightarrow u = 1$
$x = 2 \Rightarrow u = 5$

$\int_1^5 \sqrt{u} * \frac{du}{2}$
$= \frac{1}{2}\int_1^5 \sqrt{u}\,du$

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

$\frac{dV}{dt}$ $\quad$ $\sqrt{V}$ $\quad$ kx

(A) $V(t) = k\sqrt{t}$

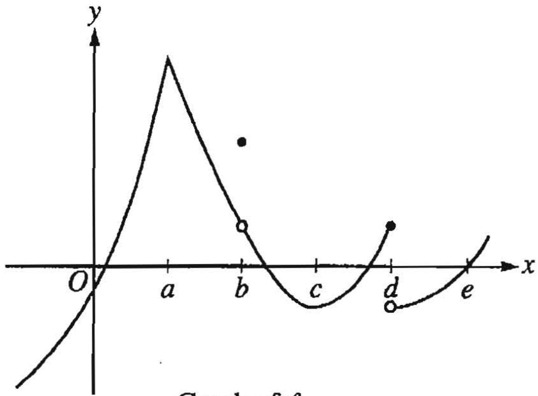
(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

$\frac{dV}{dt} = k\sqrt{V}$



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

dont lift up pencil

- removable discontinuity
- corner
- cusp
- vertical tangent
- jump

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14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

PRODUCT RULE

$u = x^2$ $v = \sin 2x$

$u' = 2x$ $v' = 2\cos 2x$ (chain rule)

$uv' + vu'$

$= x^2(2\cos 2x) + \sin 2x\,(2x)$

$= 2x^2 \cos 2x + 2x \sin 2x$

$= 2x(x \cos 2x + \sin 2x)$

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15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

decreasing when $f'(x) < 0$

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

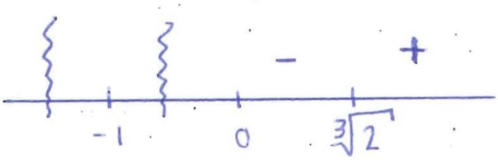
(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

* critical pts:
$f'(x) = 0$ → $x^3 - 2 = 0$ $x = \sqrt[3]{2}$
or
$f'(x) =$ DNE
→ @ $x = 0$ (0 in denom.)
→ @ $x = -1$

$f(x) = \frac{x^3}{3} - 2\ln(x)$
→ x cant be negative



16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

→ the slope @ $x = 1$

$$\frac{7-(-2)}{1-(-2)} = \frac{9}{3} = 3$$

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$f''(x) < 0$

PRODUCT RULE

$u = 2x \quad u' = 2$

$v = e^x \quad v' = e^x$

$uv' + vu' = 2xe^x + 2e^x$

$f'(x) = 2xe^x + 2e^x$

$f''(x) = 2xe^x + 2e^x + 2e^x = 2e^x(x+1+1) = 2e^x(x+2)$

$2e^x$ + + + +

$x+2$ − + + +

− + + +

−2 −1 0

$2e^x = 0$

$x + 2 = 0$

$x = -2$

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

when the derivative is less than 0.

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

aka ⇒ derivative

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$ $y' = 5 \neq 2x+3$

(B) $y = x^2 + 1$ $y' = 2x \neq 2x+3$

(C) $y = x^2 + 3x$ $y' = 2x + 3$

(D) $y = x^2 + 3x - 2$ $y' = 2x + 3$

(E) $y = 2x^2 + 3x - 3$ $y' = 4x + 3 \neq 2x+3$

Now plug in (1,2)

$2 = 1^2 + (3)(1) \Rightarrow 2 \neq 4$

$2 = 1^2 + 3(1) - 2 \Rightarrow 2 = 2$

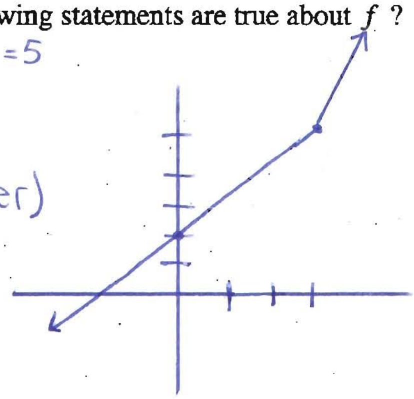
$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists. True $\lim_{x\to 3^-} = 5$ $\lim_{x \to 3^+} = 5$

II. f is continuous at $x = 3$. True

III. f is differentiable at $x = 3$. False (corner)



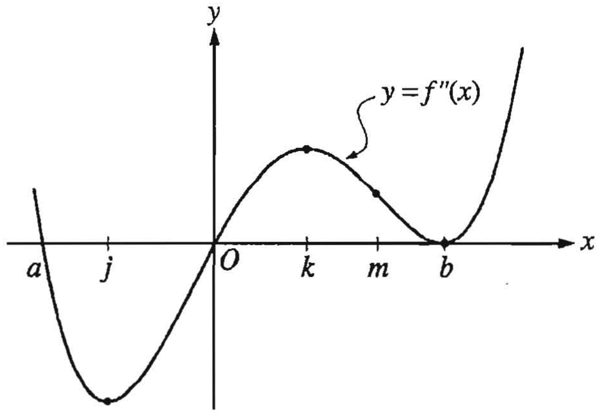
(A) None

(B) I only

(C) II only

(D) I and II only

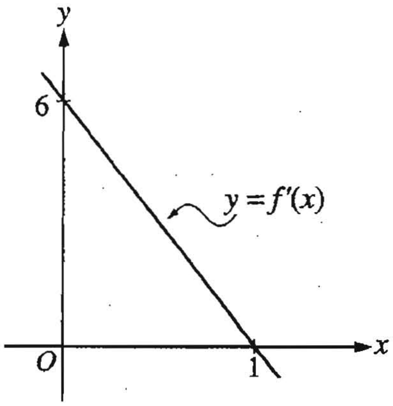
(E) I, II, and III



21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

$f''(x) = 0$ (has to go from + to − or − to +)
↳ touches x-axis
@ $x = a, 0$



22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

position ⇒ $\underbrace{\text{area under curve}}$ + initial position

$\frac{1}{2}(1)(6) \quad + \quad f(0) = 5$

$3 + 5$

8

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) =$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

* Fundamental Thm. of Calculus

(derivative of upper * plug in upper) - (deriv. of lower * plugin lower)

$2x * \sin(x^2)^3 \cancel{- 0 \sin(0)^3}$

$= 2x \sin x^6$

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

@ $x = -1$

$y = 4(-1)^3 - 5(-1) + 3$

$y = 4$

$\boxed{(-1, 4)} \Rightarrow$ Point

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5$

$\boxed{= 7} \Rightarrow$ slope

$y - 4 = 7(x + 1)$

$y - 4 = 7x + 7$

$y = 7x + 11$

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25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

velocity $= 0$

$6t^2 - 42t + 72 = 0$

$6(t^2 - 7t + 12) = 0$

$(t - 4)(t - 3)$

$t = 4 \quad t = 3$

$u = -2x$ $u' = -2$
$v = y$ $v' = \frac{dy}{dx}$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3, 2) ?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

implicit Diff.

$6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} - 2y$

$\frac{dy}{dx}(6y + 2x) = 4x - 2y$

$\frac{dy}{dx} = \frac{4x - 2y}{6y + 2x} = \frac{4(3) - 2(2)}{6(2) + 2(3)} = \frac{8}{18} = \frac{4}{9}$

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27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

(2, 1)

$f^{-1}(x) \Rightarrow x = y^3 + y$

$1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$

$\frac{dy}{dx}(3y^2 + 1) = 1$ $\frac{dy}{dx} = \frac{1}{3y^2 + 1} = \frac{1}{3(1)^2 + 1} = \frac{1}{4}$

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positive slope concave up

28. Let g be a twice-differentiable function with $\underbrace{g'(x) > 0}_{\text{positive slope}}$ and $\underbrace{g''(x) > 0}_{\text{concave up}}$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

Note:
If $f(x)$ is concave up, then the function cannot be linear.

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* Because $g'(x) > 0$ this eliminates A & B
* $m = \frac{18-12}{5-4} = 6$ $y - 12 = 6(x-4)$

$y = 6x - 12$

$g(6) = 24$

We know that $g(6) > 24$ because it is concave up.
This eliminates C & D.

END OF PART A OF SECTION I

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

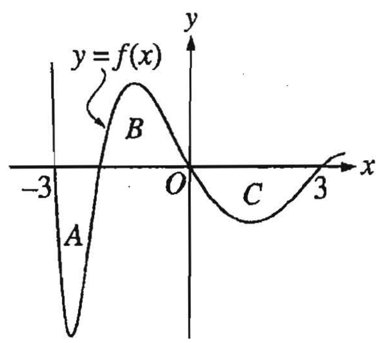
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

MATH 8
nDeriv(3+4.1cos(0.9X), X, 4) = 1.633
or
GRAPH → 2ND → TRACE → 6: → X=4



77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$?

(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

$\int_{-3}^{3} f(x) + \int_{-3}^{3} 1$

$[-2 + 2 - 2] + [x]_{-3}^{3} = 3 - (-3) = 6$ $\qquad -2 + 6 = 4$

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ \text{m}^2/\text{sec}$

(B) $0.4\pi\ \text{m}^2/\text{sec}$

(C) $4\pi\ \text{m}^2/\text{sec}$

(D) $20\pi\ \text{m}^2/\text{sec}$

(E) $100\pi\ \text{m}^2/\text{sec}$

$\frac{dr}{dt} = 0.2$

$C = 20\pi$

$C = 2\pi r$

$20\pi = 2\pi r$

$10 = r$

$\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

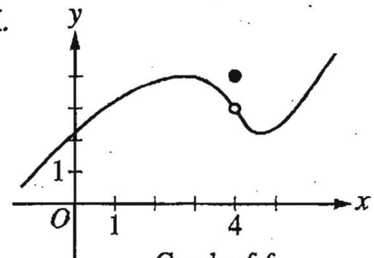
$\frac{dA}{dt} = 2\pi(10)(0.2)$

$= 4\pi \approx 12.566$

---

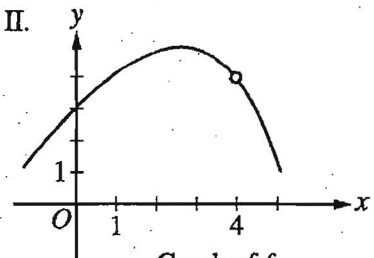
79. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.



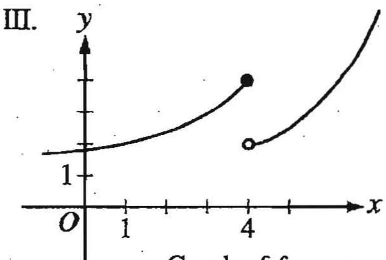
Graph of f

II.



Graph of f

III.



Graph of f

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

I: $\left.\begin{array}{l}\lim_{x\to4^+} = 3\\ \lim_{x\to4^-} = 3\end{array}\right\}$ equal

II: $\left.\begin{array}{l}\lim_{x\to4^+} = 3\\ \lim_{x\to4^-} = 3\end{array}\right\}$ equal

III: $\left.\begin{array}{l}\lim_{x\to4^+} = 1\\ \lim_{x\to4^-} = 3\end{array}\right\}$ $\neq$ limit DNE

80. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

$a = -2$
$b = 1$

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

* Mean Value Thm.

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

* Intermediate Value Thm.

if $a \le c \le b$ then $f'(a) \le f'(c) \le f'(b)$

---

81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

Not highest/
Not lowest

(A) One (B) Two (C) Three (D) Four (E) Five

* endpoints

$f(2) = \sin(2^2+1) = -.959$

$f(4) = \sin(4^2+1) = -.961$

* $f'(x) = 0$

@ $x = 2.299$

$x = 2.903$

$x = 3.401$

$x = 3.835$

6 total extrema

-1 global max

-1 global min

4 local

velocity graph ⇒ below x-axis

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

$\underbrace{1.572 \le x \le 3.514}$

when below x-axis

(B) $\int_0^8 r(t)\,dt$

(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_0^{2.667} r'(t)\,dt$

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

$$avg(t) = \frac{1}{b-a}\int_a^b f(t)\,dt$$

$$\frac{1}{3}\int_0^3 e^t + te^t$$

$$\frac{1}{3}[60.257]$$

$$= 20.086$$

---

84. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

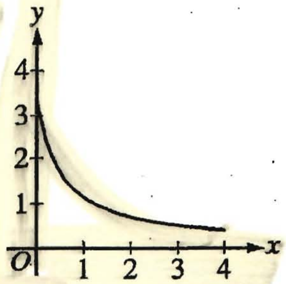
$$350 - \int_0^5 \left|-110e^{-0.4t}\right| dt$$

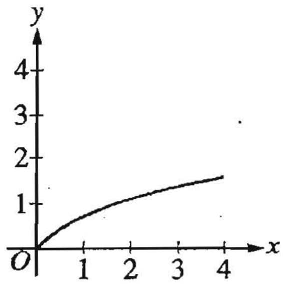
$$= 112.217$$

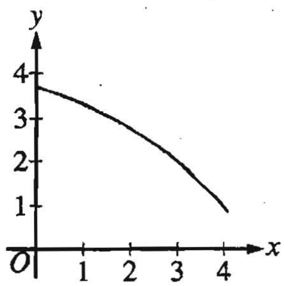
---

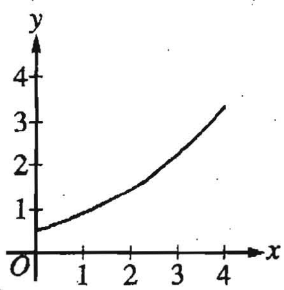
85. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

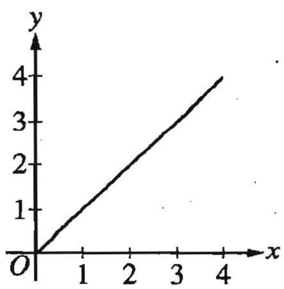
Try drawing the trapezoids & RRAM. Observe conclusions.

(A)


(B)


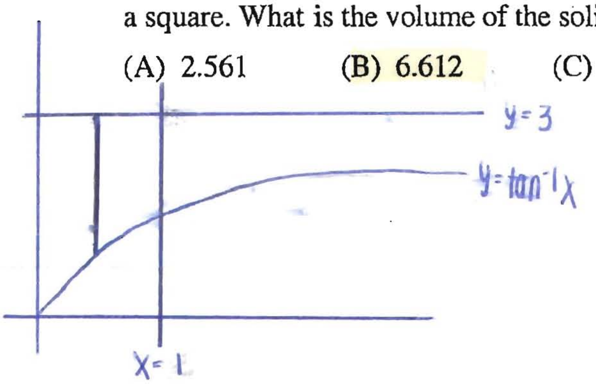
(C)


(D)


(E)


86. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 (B) 6.612 (C) 8.046 (D) 8.755 (E) 20.773



Area of a Square

x^2

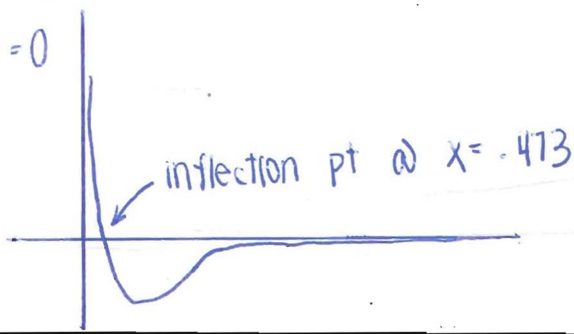
$$\int_0^1 (3-\tan^{-1}x)^2\,dx = 6.612 \text{ u}^3$$

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

inflection pt. is where $f''(x) = 0$

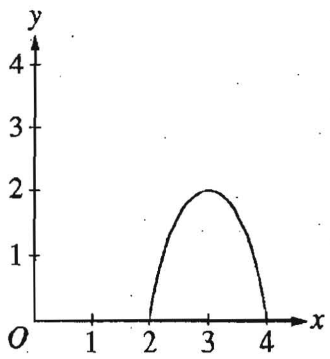
$y = \text{nDeriv}\left(\dfrac{\sqrt{x}}{1+x+x^3}, X, X\right)$



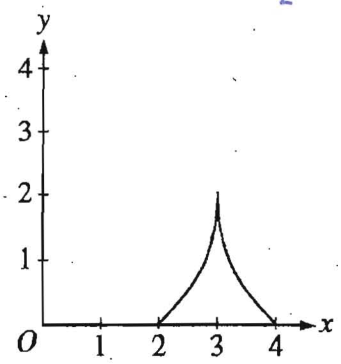
88. On the closed interval $[2, 4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

$\frac{1}{2}$ * area under curve = 1

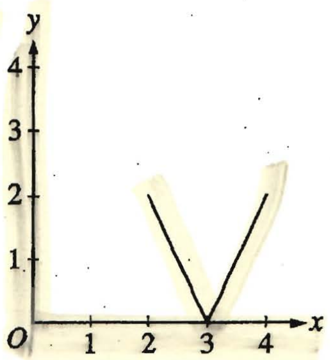
(A)



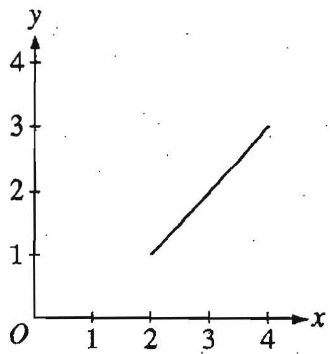
(B)



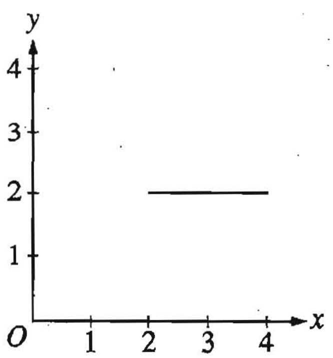
(C)



(D)



(E)



89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

$y - 3 = -5(x-2)$
$y - 3 = -5x + 10$
$y = -5x + 13$
$g(x) = x(-5x+13)$
$= -5x^2 + 13x$
$g(2) = -5(2)^2 + 13(2)$
$-20 + 26$
$g(2) = 6$ $g'(2) = -10(2) + 13$
$= -7$
$y - 6 = -7(x-2)$

90. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

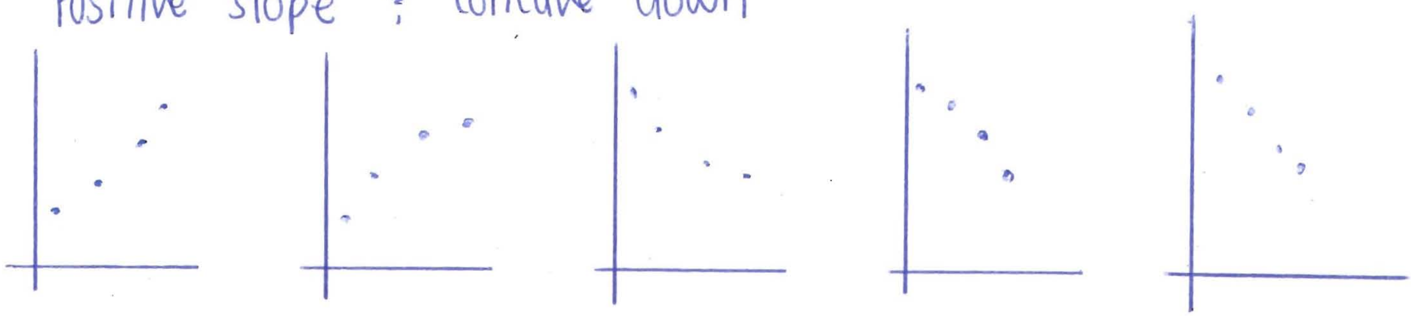
(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

positive slope $\&$ concave down



91. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

$2 + \int_1^2 \ln(1+2^t)\,dt$

$2 + 1.346$

3.346

---

92. Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

(D) $1.772 \le x \le 2.507$

(E) $2.802 \le x \le 3$

$g(x) = \sin x^2$ is below x-axis $1.772 \le x \le 2.507$

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the examination?
    (A) Casio 6300, Casio 7300, Casio 7400, Casio 7700, TI-73, TI-80, or TI-81
    (B) Casio 9700, Casio 9800, Sharp 9200, Sharp 9300, TI-82, or TI-85
    (C) Casio 9850, Casio FX 1.0, Sharp 9600, Sharp 9900, TI-83/TI-83 Plus, or TI-86
    (D) Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39G, HP 40G, HP 48 series, HP 49 series, or TI-89
    (E) Some other graphing calculator

94. During your Calculus AB course, which of the following best describes your calculator use?
    (A) I used my own graphing calculator.
    (B) I used a graphing calculator furnished by my school, both in class and at home.
    (C) I used a graphing calculator furnished by my school only in class.
    (D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
    (E) I did not use a graphing calculator.

95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?
    (A) Almost every class
    (B) About three-quarters of the classes
    (C) About one-half of the classes
    (D) About one-quarter of the classes
    (E) Seldom or never

96. During your Calculus AB course, which of the following describes the portion of testing time you were allowed to use a graphing calculator?
    (A) All or almost all of the time
    (B) About three-quarters of the time
    (C) About one-half of the time
    (D) About one-quarter of the time
    (E) Seldom or never