Big Ideas Math: Geometry

Chapter Summary

Chapter 4: Transformations

Core Vocabulary

A quantity that has both direction and magnitude and is represented in the coordinate plane by an arrow drawn from one point to another is a ***vector.***

The starting point of a vector is its ***initial point.***

The ending point of a vector is its ***terminal point.***

The horizontal change from the starting point of a vector to the ending point is its ***horizontal component.***

The vertical change from the starting point of a vector to the ending point is its ***vertical component.***

Component form is s form of a vector that combines the horizontal and vertical components.

A ***transformation*** is a function that moves or changes a figure in some way to produce a new figure.

A figure that results from the transformation of a geometric figure is its ***image.***

A ***preimage*** is a geometric figure consisting of the inputs of a transformation.

A transformation that moves every point of a figure the same distance in the same direction is a ***translation.***

A transformation that preserves length and angle measure is a ***rigid motion.***

The combination of two or more transformations to form a single transformation is a ***composition of transformations.***

A ***reflection*** is a transformation that uses a line like a mirror to reflect a figure.

A line that acts as a mirror for a reflection is a ***line of reflection.***

A transformation involving a translation followed by a reflection is a ***glide reflection.***

A figure in the plane has ***line symmetry*** when the figure can be mapped onto itself by a reflection in a line.

A line of reflection that maps a figure onto itself is a ***line of symmetry.***

A ***rotation*** is a transformation in which a figure is turned about a fixed point.

The fixed point in a rotation is the ***center of rotation.***

An ***angle of rotation*** is the angle that is formed by rays drawn from the center of rotation to a point and its image.

A figure has ***rotational symmetry*** when the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure.

The center of rotation in a figure that has rotational symmetry is the ***center of symmetry.***

Geometric figures that have the same size and shape are ***congruent figures.***

A ***congruence transformation*** is a transformation that preserves length and angle measure.

A transformation in which a figure is enlarged or reduced with respect to a fixed point is a ***dilation.***

The fixed point in a dilation is the ***center of dilation.***

The ratio of the lengths of the corresponding sides of the image and the preimage of a dilation is the ***scale factor.***

A dilation in which the scale factor is greater than 1 is an ***enlargement.***

A dilation in which the scale factor is greater than 0 and less than 1 is a ***reduction.***

A ***similarity transformation*** is a dilation or a composition of rigid motions and dilations.

Geometric figures that have the same shape but not necessarily the same size are ***similar figures.***

Standards

Common Core:
HSG-CO.A.2, HSG-CO.A.3, HSG-CO.A.4, HSG-CO.A.5, HSG-CO.B.6, HSG-SRT.A.1a, HSG-SRT.A.1b, HSG-SRT.A.2, HSG-MG.A.3

Essential Questions

How can you translate a figure in a coordinate plane?

How can you reflect a figure in a coordinate plane?

How can you rotate a figure in a coordinate plane?

What conjectures can you make about a figure reflected in two lines?

What does it mean to dilate a figure?

When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure?

Postulates

4.1 Translation Postulate
A translation is a rigid motion.

4.2 Reflection Postulate
A reflection is a rigid motion.

4.3 Rotation Postulate
A rotation is a rigid motion.

Learning Goals

Perform translations.

Perform compositions.

Solve real-life problems involving compositions.

Perform reflections.

Perform glide reflections.

Identify lines of symmetry.

Solve real-life problems involving reflections.

Perform rotations.

Perform compositions with rotations.

Identify rotational symmetry.

Identify congruent figures.

Describe congruence transformations.

Use theorems about congruence transformations.

Identify and perform dilations.

Solve real-life problems involving scale factors and dilations.

Perform similarity transformations.

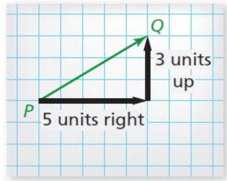
Describe similarity transformations.

Prove that figures are similar.

Core Concept

Vectors

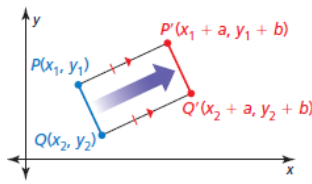
The diagram shows a vector. The initial point, or starting point, of the vector is P, and the terminal point, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The horizontal component of $\overrightarrow{PQ}$ is 5, and the vertical component is 3. The component form of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.



Translations

A translation moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q', so that one of the following statements is true.

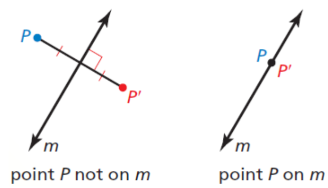
- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



Reflections

A reflection is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the line of reflection. A reflection in a line m maps every point P in the plane to a point P', so that for each point one of the following properties is true.

- If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



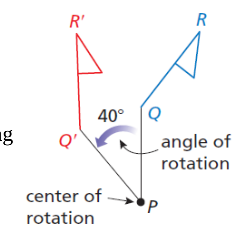
Coordinate Rules for Reflections

- If (a, b) is reflected in the x-axis, then its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, then its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, then its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, then its image is the point $(-b, -a)$.

Rotations

A rotation is a transformation in which a figure is turned about a fixed point called the center of rotation. Rays drawn from the center of rotation to a point and its image form the angle of rotation. A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true.

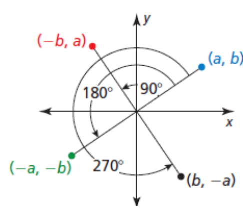
- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then $Q = Q'$.



Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

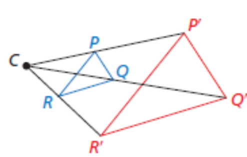
- For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.



Dilations

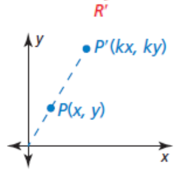
A dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the center of dilation and a scale factor k, which is the ratio of the lengths of the corresponding sides of the image and the preimage. A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.



Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.



$(x, y) \rightarrow (kx, ky)$

Theorems

4.1 Composition Theorem
The composition of two (or more) rigid motions is a rigid motion.

4.2 Reflections in Parallel Lines Theorem
If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation. If A'' is the image of A, then

1. $\overline{AA''}$ is perpendicular to k and m, and
2. $AA'' = 2d$, where d is the distance between k and m.

4.3 Reflections in Intersecting Lines Theorem
If lines k and m intersect at point P, then a reflection in line k followed by a reflection in line m is the same as a rotation about point P. The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by lines k and m.

Games

- A New You

This is available online in the *Game Closet* at www.bigideasmath.com.

Additional Review

- Line Symmetry, *p. 185*
- Rotational Symmetry, *p. 193*
- Identifying Congruent Figures, *p. 200*
- Describing a Congruence Transformation, *p. 201*
- Dilations and Scale Factor, *p. 208*
- Negative Scale Factors, *p. 210*
- Similarity Transformations, *p. 216*

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 4: Rotational Doors STEM Video is available online at www.bigideasmath.com.