Big Ideas Math: Algebra 2

Chapter Summary

Chapter 4: Polynomial Functions

Standards
Common Core: HSN-CN.C.8, HSN-CN.C.9, HSA-SSE.A.2, HSA-APR.A.1, HSA-APR.B.2, HSA-APR.B.3, HSA-APR.C.4, HSA-APR.C.5, HSA-APR.D.6, HSA-CED.A.2, HSF-IF.B.4, HSF-IF.C.7c, HSF-BF.A.1a, HSF-BF.B.3

Learning Goals

Identify polynomial functions.

Graph polynomial functions using tables and end behavior.

Add and subtract polynomials.

Multiply polynomials.

Use Pascal's Triangle to expand binomials.

Use long division to divide polynomials by other polynomials.

Use synthetic division to divide polynomials by binomials of the form $x - k$.

Use the Remainder Theorem.

Factor polynomials.

Use the Factor Theorem.

Find solutions of polynomial equations and zeros of polynomial functions.

Use the Rational Root Theorem.

Use the Irrational Conjugates Theorem.

Use the Fundamental Theorem of Algebra.

Find conjugate pairs of complex zeros of polynomial functions.

Use Descartes's Rule of Signs.

Describe transformations of polynomial functions.

Write transformations of polynomial functions.

Use x-intercepts to graph polynomial functions.

Use the Location Principle to identify zeros of polynomial functions.

Find turning points and identify local maximums and local minimums of graphs of polynomial functions.

Identify even and odd functions.

Write polynomial functions for sets of points.

Write polynomial functions using finite differences.

Use technology to find models for data sets.

Essential Questions

What are some common characteristics of the graphs of cubic and quartic polynomial functions?

How can you cube a binomial?

How can you use the factors of a cubic polynomial to solve a division problem involving the polynomial?

How can you factor a polynomial?

How can you determine whether a polynomial equation has a repeated solution?

How can you determine whether a polynomial equation has imaginary solutions?

How can you transform the graph of a polynomial function?

How many turning points can the graph of a polynomial function have?

How can you find a polynomial model for real-life data?

Core Vocabulary

A ***polynomial*** is a monomial or a sum of monomials.

A ***polynomial function*** is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers.

The ***end behavior*** of a function's graph is the behavior of the graph as x approaches positive infinity or negative infinity.

Pascal's Triangle is a triangular array of numbers such that the numbers in the nth row are the coefficients of the terms in the expansion of $(a + b)^n$ for whole number values of n.

Polynomial long division is a method to divide a polynomial $f(x)$ by a nonzero divisor $d(x)$ to yield a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

A shortcut method to divide a polynomial by a binomial of the form $x - k$ is called ***synthetic division***.

A factorable polynomial with integer coefficients is ***factored completely*** when it is written as a product of unfactorable polynomials with integer coefficients.

A method of factoring a polynomial by grouping pairs of terms that have a common monomial factor is called ***factor by grouping***.

An expression of the form $au^2 + bu + c$, where u is an algebraic expression, is said to be in ***quadratic form***.

A solution of an equation that appears more than once is a ***repeated solution***.

Pairs of complex numbers of the forms $a + bi$ and $a - bi$, where $b \neq 0$, are called ***complex conjugates***.

The y-coordinate of a turning point is a ***local maximum*** of the function when the point is higher than all nearby points.

The y-coordinate of a turning point is a ***local minimum*** of the function when the point is lower than all nearby points.

A function f is an ***even function*** when $f(-x) = f(x)$ for all x in its domain.

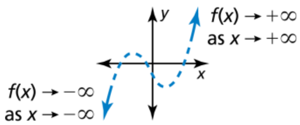
A function f is an ***odd function*** when $f(-x) = -f(x)$ for all x in its domain.

The differences of consecutive y-values in a data set when the x-values are equally spaced are called ***finite differences***.

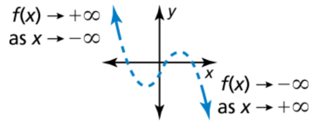
Core Concept

End Behavior of Polynomial Functions

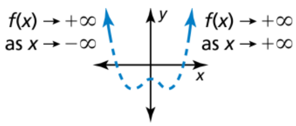
Degree: odd
Leading coefficient: positive



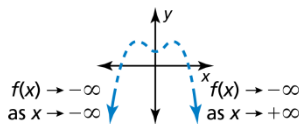
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive



Degree: even
Leading coefficient: negative



The Fundamental Theorem of Algebra

Theorem

- If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary

- If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients.

- The number of *positive real zeros* of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of *negative real zeros* of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Special Product Patterns

Sum and Difference

- $(a + b)(a - b) = a^2 - b^2$

Square of a Binomial

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

Cube of a Binomial

- $(a + b)^3 = a^3 + 3a^2b + 3ab_2 + b^3$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Special Factoring Patterns

Sum of Two Cubes

- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Two Cubes

- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a + b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a + b)^0 =$	1	1
1st row	1	$(a + b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a + b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a + b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a + b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a + b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

Core Concept

Transformations

Transformation	$f(x)$ Notation
Horizontal Translation Graph shifts left or right.	$f(x - h)$
Vertical Translation Graph shifts up or down.	$f(x) + k$
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$

Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function. The following statements are equivalent.

- **Zero**: k is a zero of the polynomial function f.
- **Factor**: $x - k$ is a factor of the polynomial $f(x)$.
- **Solution**: k is a solution (or root) of the polynomial equation $f(x) = 0$.
- ***x*-Intercept**: If k is a real number, then k is an x-intercept of the graph of the polynomial function f. The graph of f passes through $(k, 0)$.

The Location Principle

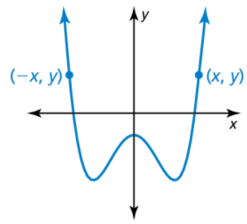
If f is a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b.

Turning Points of Polynomial Functions

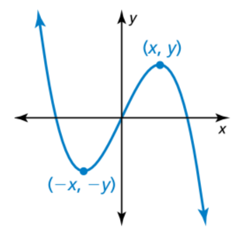
1. The graph of every polynomial function of degree n has *at most* $n - 1$ turning points.
2. If a polynomial function has n distinct real zeros, then its graph has *exactly* $n - 1$ turning points.

Even and Odd Functions

- A function f is an even function when $f(-x) = f(x)$ for all x in its domain. The graph of an even function is *symmetric about the y-axis*.



- For an even function, if (x, y) is on the graph, then $(-x, y)$ is also on the graph.
- A function f is an odd function when $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is *symmetric about the origin*. One way to recognize a graph that is symmetric about the origin is that it looks the same after a 180° rotation about the origin.



- For an odd function, if (x, y) is on the graph, then $(-x, -y)$ is also on the graph.

Properties of Finite Differences

1. If a polynomial function $f(x)$ has degree n, then the nth differences of function values for equally-spaced x-values are nonzero and constant.
2. Conversely, if the nth differences of equally-spaced data are nonzero and constant, then the data can be represented by a polynomial function of degree n.

Games

- A Dicey Polynomial Situation
- Equation Tic-Tac-Toe
- Transform Me
- Make My Team
- Polynomial Tic-Tac-Toe

These are available online in the *Game Closet* at www.bigideasmath.com.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 4: Quonset Huts STEM Video is available online at www.bigideasmath.com.

Additional Review

- Common Polynomial Functions, *p. 158*
- Graphing Polynomial Functions, *p. 160*
- Operations with Polynomials, *p. 166*
- Polynomial Long Division, *p. 174*
- Synthetic Division, *p. 175*
- Factoring Polynomials, *p. 180*
- Writing Transformed Polynomial Functions, *p. 207*
- Writing Polynomial Functions for Data Sets, *p. 220*