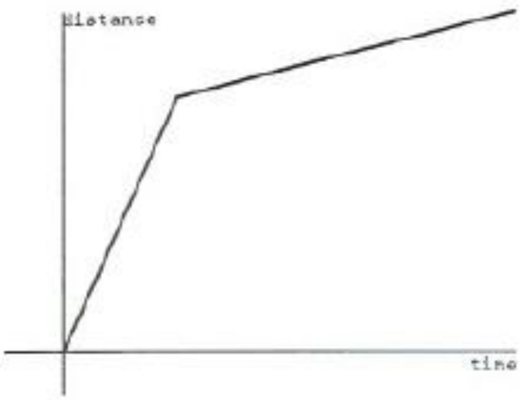
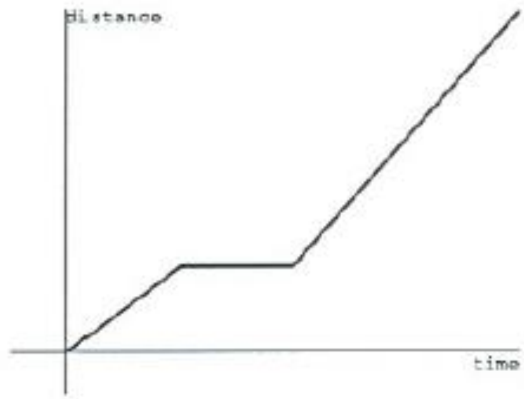
Biking to school[1]

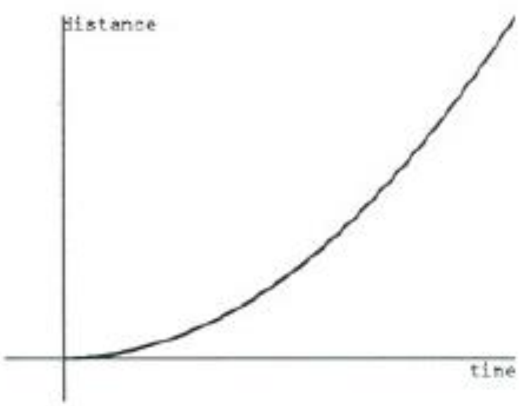
Terry usually rides a bicycle to school. Below are four graphs and three explanations. Match each explanation with a graph, and write an explanation for the remaining graph.



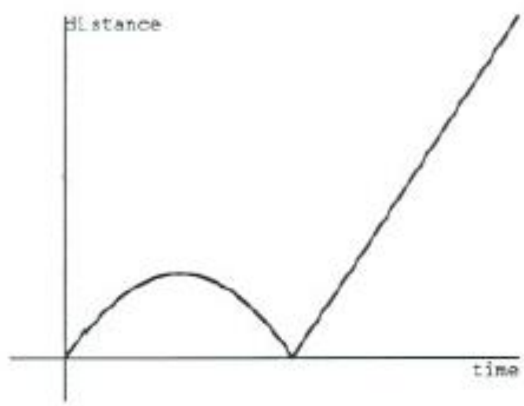
A



B



C



D

1. "I had just left home when I realized we have gym today, and I had forgotten my gym clothes. So I went back home and then I had to hurry to be on time."
2. "I always start off very calmly. After a while I speed up, because I don't like to be late."
3. "I went on my motor bike this morning, very quickly. After a while, I ran out of gas. I had to walk the rest of the way and was just on time."
4.

AP CALCULUS Name:________________________ Period: _____

INTRO TO PARTICLE MOTION PRACTICE (No calculator)

1) A particle travels along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = t^3 - 9t^2 + 24t + 2$. For what value(s) of t is the velocity equal to zero?

A) $t = 4$, only
B) $t = 2$, only
C) $t = 0$ and $t = 3$
D) $t = 3$, only
E) $t = 2$ and $t = 4$

2) A particle moves on the x-axis so that its position is given by $x(t) = t^4 - 6t^2 + 8$ for $t \geq 0$. For what times t is the velocity of the particle increasing?

A) $t > 0$
B) $0 < t < \sqrt{3}$
C) $t > \sqrt{3}$
D) $0 < t < 1$
E) $1 < t < \sqrt{3}$

3) The position of a particle moving on a horizontal axis for time t, where $t \geq 0$, is $S(t) = 3 \sin \frac{1}{2}t + 1$. What is the average velocity of the particle for $0 \leq t \leq \frac{3\pi}{2}$?

A) $\frac{\pi}{\sqrt{2}}$
B) $\frac{\sqrt{2}}{\pi}$
C) $\frac{3\sqrt{2}}{\pi}$
D) $\frac{-\sqrt{2}}{\pi}$
E) $-\frac{\pi}{\sqrt{2}}$

4) What is the maximum acceleration of a particle on the interval $0 \leq t \leq 3$ if its position is given by $s(t) = t^4 - 4t^3$?

A) 36
B) -16
C) 0
D) -12
E) 24

5) The table below shows the position of a particle, S, at various times, t, as it moves along a straight line.

t (sec)	1.0	1.4	1.8	2.2	2.6
s (ft)	6.0	7.0	10.0	15.0	21.0

What is an estimated value of the velocity of the particle at time $t = 2$?

A) 15 ft/sec
B) 12.5 ft/sec
C) 20 ft/sec
D) 10 ft/sec
E) 5 ft/sec

6) If the position of a particle moving on the x-axis at any time t is given by $x(t) = 2t^3 - 3t^2$, what is the average acceleration of the particle for $0 \leq t \leq 3$?

A) 15
B) 18
C) 8
D) 9
E) 12

7) A particle moves along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = 2t + \sin(\pi t)$. What is the acceleration of the particle at time $t = \frac{3}{2}$?

A) $-\pi^2$
B) 2
C) π
D) π^2
E) 0

8) A particle moves along the x-axis so that is position at any time t is given by $x(t) = t^3 - 6t^2 + 9t + 12$. During what times is the speed of the particle increasing?

A) $t < 1$ or $2 < t < 3$
B) $1 < t < 2$ or $t > 3$
C) $t < 2$ or $t > 3$
D) $1 < t < 3$
E) $t < 1$ or $t > 3$

9) A particle moves along a coordinate line so that its position is given by $S(t) = 2\sin\frac{1}{2}t + \frac{1}{2}\cos 2t$ for $0 \le t \le 2\pi$. What is the acceleration of the particle at $t = \pi$?

A) $-\frac{3}{2}$
B) $-\frac{1}{2}$
C) 1
D) $-\frac{5}{2}$
E) $\frac{5}{2}$

10) A particle moves along the x-axis in such a way that its position at any time t is given by $x(t) = t^4 - 8t^3 + 18t^2 + 2$ for $t > 0$. At what time is acceleration of the particle equal to 36?

A) 3
B) 4
C) 12
D) 6
E) 2

11) A particle moves on the x-axis such that its position at any time $t > 0$ is given by $x(t) = t^3 - 9t^2 + 24t$. What is the velocity of the particle when its acceleration is zero?

A) 105
B) 24
C) -3
D) 3
E) 0

12) A particle moves along a horizontal axis so that its position is defined by $S(t) = 4\cos\frac{\pi}{2}t$ for $0 \le t \le 5$. What is the velocity of the particle at the time its acceleration is first equal to zero?

A) 2π
B) -2π
C) -4π
D) 4π
E) $-\pi^2$

13) A particle moves along a horizontal coordinate line so that its position at time t, $0 \le t \le 4$ is given by $S(t) = t^4 - \frac{16}{3}t^3 + 8t^2 + 1$. For what times t is the velocity of the particle decreasing?

A) $\frac{2}{3} < t < 2$
B) $t > \frac{2}{3}$
C) $0 < t < 2$
D) $0 < t < 4$
E) $2 < t < 4$

14) The table below shows velocity of a particle at various times t of a particle that moves along a horizontal line.

t (sec)	0.5	1.0	1.5	2.0
v (m/sec)	8.3	9.2	9.8	10.6

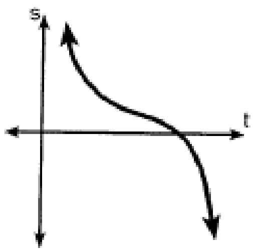
What is an approximate value of the acceleration of the particle at time $t = 2$?

A) 1.2 ft/sec^2
B) -0.8 ft/sec^2
C) 1.6 ft/sec^2
D) -1.6 ft/sec^2
E) 1.8 ft/sec^2

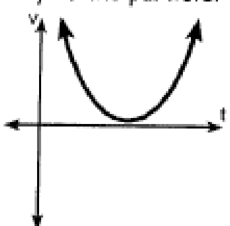
AP CALCULUS Name:_________________________ Period: ____

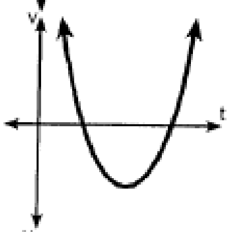
PARTICLE MOTION GRAPH PRACTICE

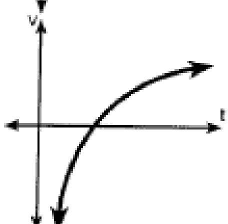
1) The position function, $S(t)$, of a particle moving on a horizontal line is given below.

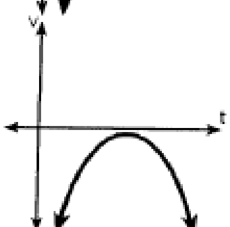


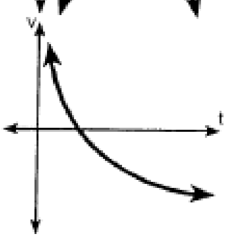
Which of the accompanying graphs represents the velocity of the particle?

A) 

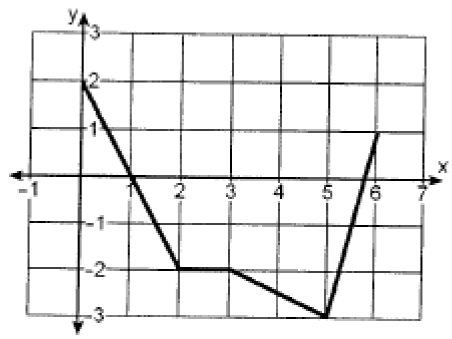
B) 

C) 

D) 

E) 

2) The graph below shows the velocity for a particle along the x-axis over the time interval $0 \le t \le 6$.

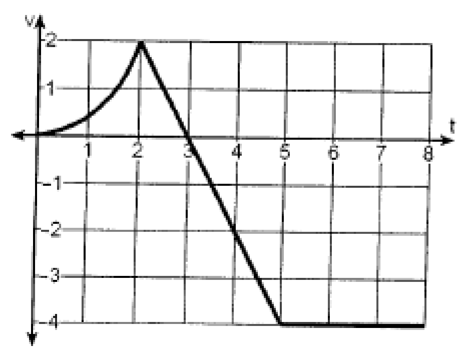


At what time does the particle reach its maximum speed?

A) 5
B) 1
C) 2
D) 0
E) 6

Questions 3 through 7 refer to the following:

The graph below shows the velocity of a particle that moves on a horizontal line for time $0 \le t \le 8$.



3) Using the graph shown, how many times does the particle change directions for $0 < t < 8$?

A) 1
B) 2
C) 3
D) 0
E) 5

4) According to the graph shown, during what interval of time is the particle slowing down?

A) $5 < t < 8$
B) $0 < t < 2$
C) $3 < t < 8$
D) $2 < t < 5$
E) $2 < t < 3$

5) According to the graph shown, how many times for $0 < t < 8$ is acceleration *not* defined?

A) 1
B) 2
C) 3
D) 4
E) 0

6) Using the graph shown, what is the average acceleration during the time interval $2 \le t \le 5$?

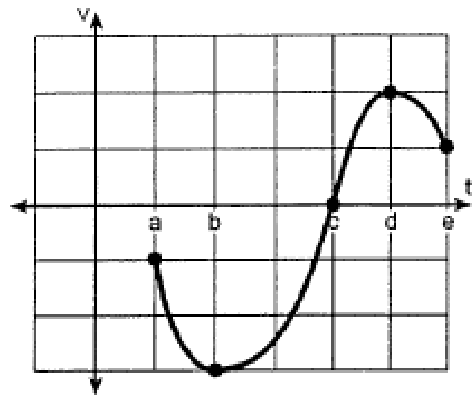
A) $\frac{1}{2}$
B) 2
C) -1
D) -2
E) $-\frac{1}{2}$

7) At what time does the particle in the graph shown *first* reach its maximum speed?

A) 8
B) 2
C) 3
D) 4
E) 5

Questions 8 through 13 refer to the following:

The graph below shows the velocity of a particle as a function of time t, $a \le t \le e$.



8) According to the graph shown, for what value of t i the particle's position *farthest* to the left?

A) a
B) b
C) c
D) d
E) e

9) According to the graph, for what time t does the particle reach maximum speed?

A) a
B) b
C) c
D) d
E) e

10) According to the graph shown, for what times, t, is the particle speeding up?

A) $b < t < e$ or $d < t < e$
B) $d < t < e$, only
C) $c < t < d$, only
D) $a < t < b$, only
E) $a < t < b$ or $c < t < d$

11) Using the graph shown, for what times, *t*, is the particle's speed decreasing?

A) $b < t < c$ or $d < t < e$
B) $d < t < e$, only
C) $b < t < c$, only
D) $a < t < b$ or $c < t < d$
E) $a < t < b$, only

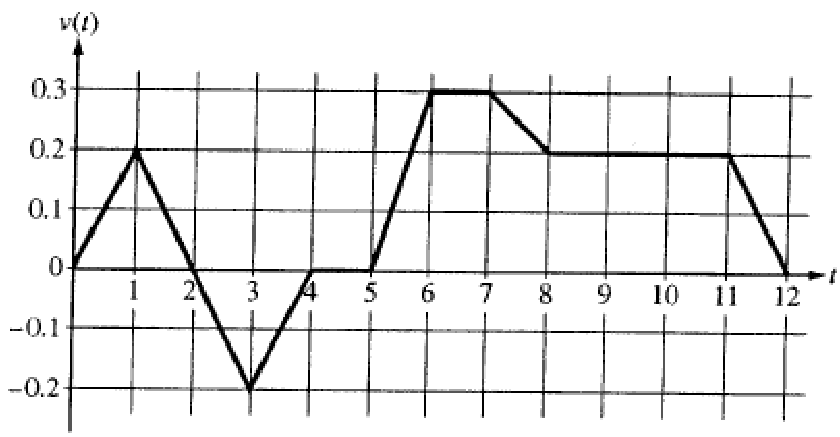
12) According to the graph shown, which statement *best* describes the motion of the particle for $b < t < c$?

A) The particle is moving to the left at a constant speed.
B) The particle is moving to the left with decreasing speed.
C) The particle is moving to the left with increasing speed.
D) The particle is moving to the right with increasing speed.
E) The particle is moving to the right with decreasing speed.

13) Using the information shown in the graph, at what time *t* does the particle reverse direction?

A) *a*
B) *b*
C) *c*
D) *d*
E) *e*

14.

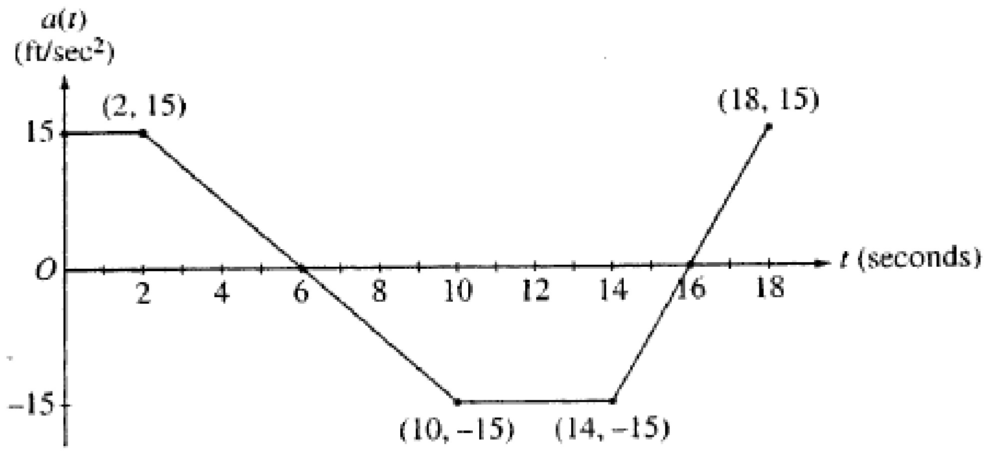


Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

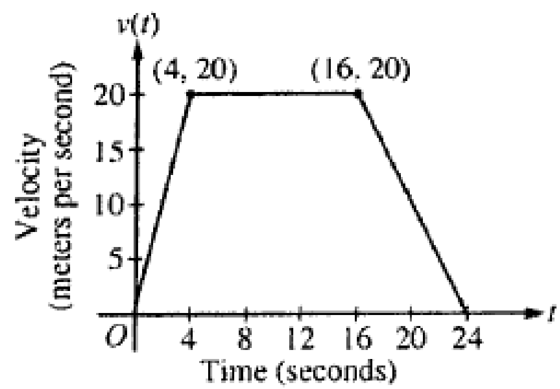
15.



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

16.



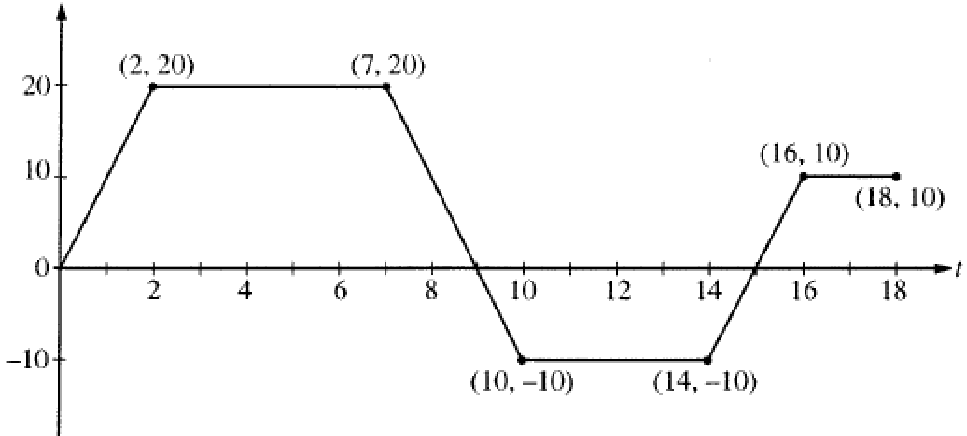
A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

17.



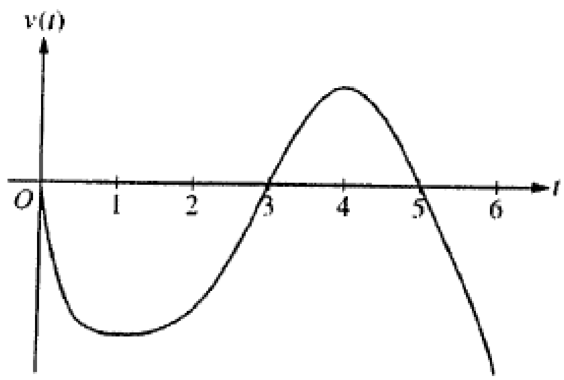
Graph of v

A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(d) Write expressions for the squirrel's acceleration $a(t)$ and velocity $v(t)$, that are valid for the time interval $7 < t < 10$.

18.



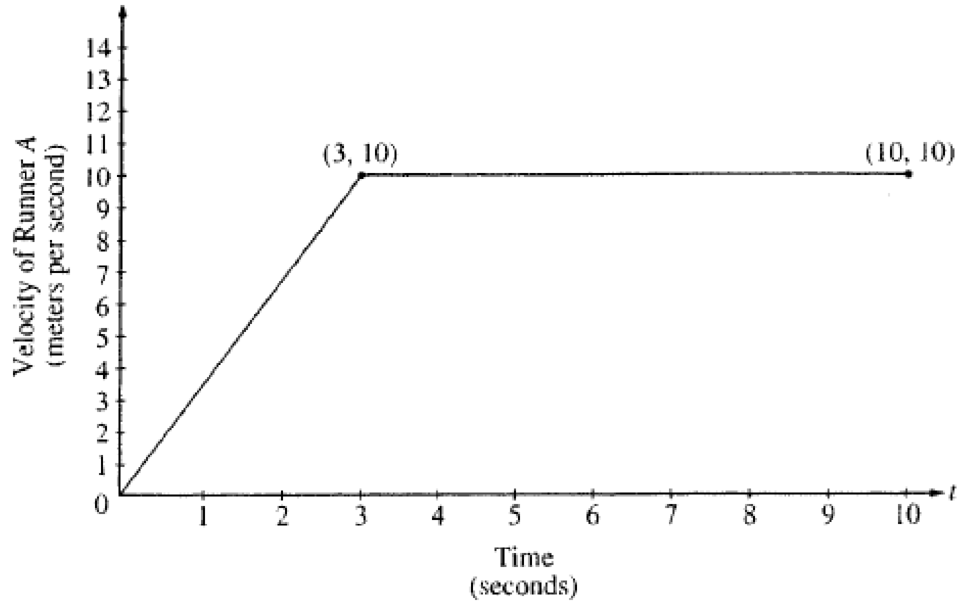
Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

19.

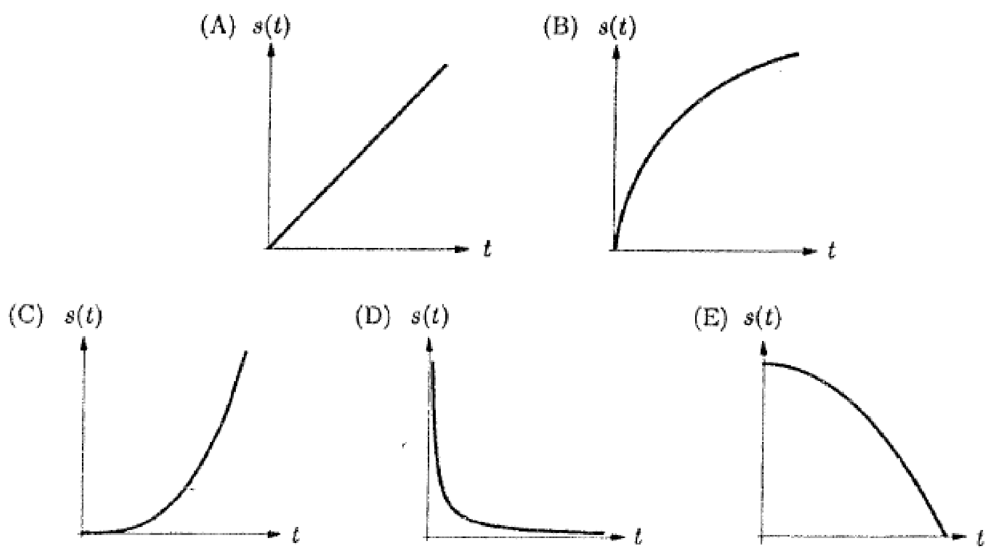


Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

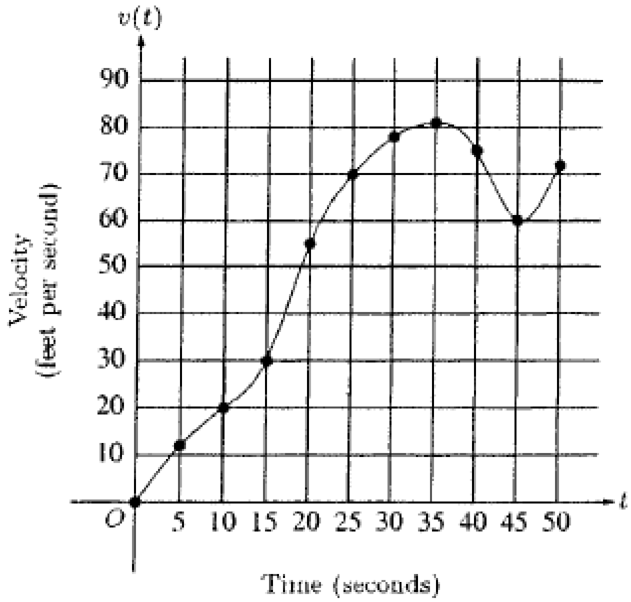
(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

20. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?



21.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

AP CALCULUS
VELOCITY & SPEED PRACTICE

Name ________________________________

Directions: No calculator! Use your notes!

1) A particle moves along a coordinate line in such a way that is position is given by $S(t) = 2\sqrt{t}(2 - 2t + t^2)$ for $t \geq 0$. For what times is the particle moving to the left?

A) $0 < t < 2$
B) The particle never moves to the left.
C) $1 < t < 2$
D) $2 < t < 5$
E) $t > 5$

2) A particle moves along the x-axis so that its position at time t is given by $x(t) = 2t^2 - 12t + 9$. For what value of t is the particle at rest?

A) 1
B) 4
C) 3
D) 0
E) 9

3) A particle moves along the x-axis in such a way that its position is given by $x(t) = 2t^3 + 24t - 4\cos\left(\frac{\pi}{2}t\right)$ for $t > 0$. When is the particle moving to the left?

A) $t > 0$
B) $0 < t < 2$
C) The particle never moves to the left.
D) $0 < t < \frac{\pi}{2}$
E) $t > \frac{\pi}{2}$

4) A particle moves along a horizontal axis so that its position is given by $x(t) = 4t^5 - 5t^3$ for any time t. How many times does the particle change direction?

A) 1
B) 2
C) 3
D) 0
E) 5

5) A particle moves along the x-axis so that its position at any time t is given by $x(t) - t^4 - 6t^2 - 3$. Which of the following *best* describes the motion of the particle for $0 < t < 1$?

A) Moving to the right and speeding up.
B) Moving to the left and speeding up.
C) Moving to the right and slowing down.
D) Moving to the left at a constant speed.
E) Moving to the left and slowing down.

6) A particle moves along the x-axis so that its position is given by $x(t) = 2\cos(2t)$ for $0 \le t \le \pi$. Which statement *best* describes the motion of the particle for $\frac{\pi}{4} < t < \frac{\pi}{2}$?

A) moving to the right at increasing speed
B) moving to the left at decreasing speed
C) moving to the left at constant speed
D) moving to the right at decreasing speed
E) moving to the left at increasing speed

7) A particle moves along the x-axis so that is position at any time t is given by $x(t) = t^3 - 6t^2 + 9t + 12$. During what times is the speed of the particle increasing?

A) $t < 1$ or $t > 3$
B) $1 < t < 2$ or $t > 3$
C) $t < 1$ or $2 < t < 3$
D) $t < 2$ or $t > 3$
E) $1 < t < 3$

8) A particle moves along the x-axis so that its position is given by $x(t) = 4t^3 - 3t^2$ for any time $t \ge 0$. During what time interval is the particle's position to the left of zero?

A) $0 < t < \frac{3}{4}$
B) $0 < t < \frac{4}{3}$
C) $\frac{1}{2} < t < \frac{3}{4}$
D) $1 < t < \frac{3}{4}$
E) $0 < t < \frac{1}{2}$

9) A particle moves on a straight line in such a way that its distance at any time t from a fixed point on the line is given by $S(t) = 4t - 3t^2$. What is the total distance traveled by the particle between $t = 0$ and $t = 2$?

A) $\frac{16}{3}$
B) $\frac{20}{3}$
C) $\frac{22}{3}$
D) 6
E) 4

10. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$.

A. When is the particle at rest?
B. When is the particle moving to the right?
C. Find all values of t for which the particle moving to the left.
D. When does the particle change direction?
E. What is the velocity of the particle at $t = 3$?
F. What is the total distance that the particle traveled from time $t = 0$ to $t = 3$?

11. A particle moves along a line such that its position function is given by $s(t) = t^3 - 12t + 3$ for the time interval [0, 3]. Find the following:

A. When the particle is at rest, moving to the left, and to the right.
B. The intervals where the particle is speeding up and slowing down.
C. Draw a picture that illustrates the motion of the particle.
D. The total distance that the particle travels over the indicated time interval.

AP Calculus
Miss Brown

Name ______________________
Date ______________________

Particle Motion -- With your Graphing Calculator!

1. (2005-B) A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$.

A. Find the acceleration of the particle at time $t = 4$.
B. Is the particle speeding up or slowing down at $t = 4$? Explain.
C. Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?
D. Find the average acceleration of the particle over the interval $0 \le t \le 2$.

2. (2004) A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$.

A. Find the acceleration of the particle at time $t = 2$.
B. Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
C. Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

3. (2003) A particle moves along the x-axis so that its velocity at any time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$.

A. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

B. Find all times in the interval $0 \le t \le 3$ when the particle changes direction. Justify your answer.

C. If the particle starts at the origin at $t = 0$, on which side of the origin will the particle be at $t = 2$? Justify your answer.

4. (2002-B) A particle moves along the x-axis so that its velocity at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

A. At what time is the particle at rest? Justify your answer.

B. During what intervals of time is the particle moving to the left? Give a reason for your answer.

C. What is the acceleration of the particle at time $t = 2$? Is the particle's speed increasing or decreasing at time $t = 2$? Justify your answer.

5. (2002) An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \ge 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

A. What is the acceleration of the object at time $t = 4$?

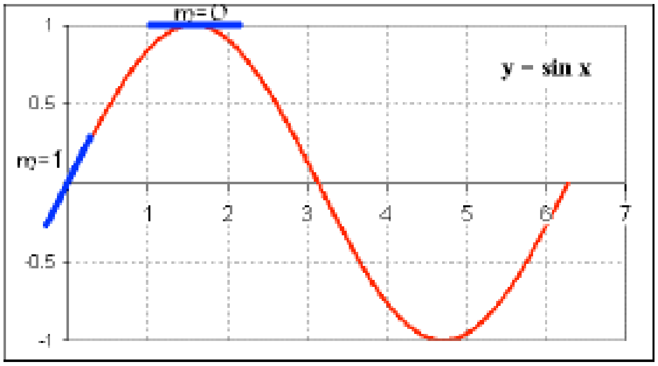
B. Consider the following two statements.

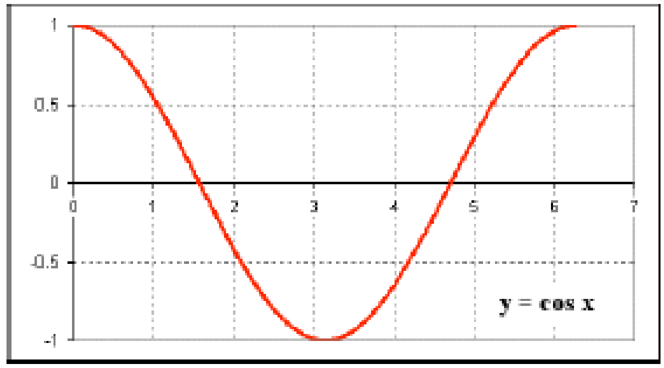
Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

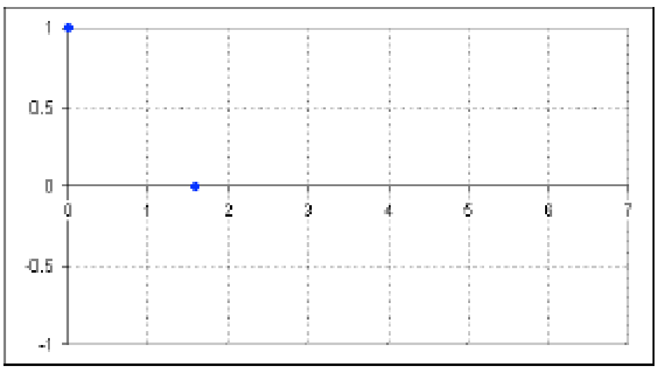
Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

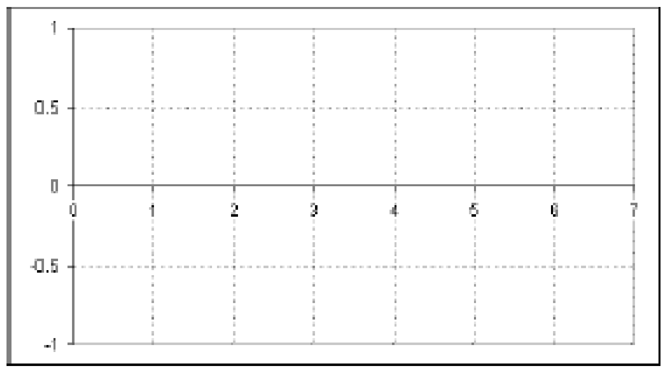
In-Class Problems
Derivatives of Trig Functions

1. Carefully sketch the derivative of the following two functions by approximating the slope of each function at a number of points. (Note: the dashed boxes are one unit wide and half a unit tall.)









2. Given the graphs you have put together, what would you *guess* are the derivatives of sine and cosine?

a. $\dfrac{d}{dx}\sin x =$ ____________

b. $\dfrac{d}{dx}\cos x =$ ____________

3. Assuming that your guesses are correct (they should be), use the quotient rule to find the derivatives of the other basic trigonometric functions.

a. $\tan x = \dfrac{\sin x}{\cos x}$ so $\dfrac{d}{dx}\tan x =$ ____________

b. $\cot x = \dfrac{\cos x}{\sin x}$ so $\dfrac{d}{dx}\cot x =$ ____________

c. $\sec x = \dfrac{1}{\cos x}$ so $\dfrac{d}{dx}\sec x =$ ____________

d. $\csc x = \dfrac{1}{\sin x}$ so $\dfrac{d}{dx}\csc x =$ ____________

4. What limit would you need to evaluate in order to *prove* the guess that you made in 2a?

Handout - Derivative - Chain Rule

Power-Chain Rule a, b are constants.

Function	Derivative	
$y = a \cdot x^n$	$\frac{dy}{dx} = a \cdot n \cdot x^{n-1}$	Power Rule
$y = a \cdot u^n$	$\frac{dy}{dx} = a \cdot n \cdot u^{n-1} \cdot \frac{du}{dx}$	**Power-Chain Rule**

Ex1a. Find the derivative of $y = 8(6x + 21)^8$

Answer: $\boldsymbol{y' = 384(6x + 21)^7}$

$$a = 8, \quad n = 8$$
$$u = 6x + 21 \quad \Rightarrow \frac{du}{dx} = 6$$
$$\Rightarrow y' = 8 \cdot 8 \cdot (6x + 21)^7 \cdot 6$$

Ex1b. Find the derivative of $y = 8\left(4x^2 + 7x + 28\right)^4$

Answer: $\boldsymbol{y' = 32(8x + 7)\left(4x^2 + 7x + 28\right)^3}$

$$a = 8, \quad n = 4$$
$$u = 4x^2 + 7x + 28 \quad \Rightarrow \frac{du}{dx} = 8x + 7$$
$$\Rightarrow y' = 8 \cdot 4 \cdot \left(4x^2 + 7x + 28\right)^3 \cdot (8x + 7)$$

Ex1c. Find the derivative of $y = 2\sqrt{6x^2 + 4x + 26}$

Answer: $\boldsymbol{y' = \frac{12x + 4}{\sqrt{6x^2 + 4x + 26}}}$

$$a = 2, \quad n = \frac{1}{2}$$
$$u = 6x^2 + 4x + 26 \quad \Rightarrow \frac{du}{dx} = 12x + 4$$
$$\Rightarrow y' = 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{6x^2+4x+26}} \cdot (12x + 4)$$

Exercises

Find the derivatives of the expressions

a) $5(9x+25)^8$

b) $7(2x+24)^8$

c) $2\left(4x^2+4x+21\right)^9$

d)$6\left(7x^2+4x+22\right)^4$

e) $7\left(7x^2+9x+24\right)^{13/3}$

f)$3\left(7x^2+4x+29\right)^{22/3}$

g) $5\sqrt{x^2+8x+25}$

h)$7\sqrt{7x^2+3x+24}$

i) $\frac{8}{\sqrt{3x^2+3x+22}}$

j)$\frac{3}{\sqrt{2x^2+2x+22}}$

Answers a) $360(9x+25)^7$; b) $112(2x+24)^7$;
c) $18(8x+4)\left(4x^2+4x+21\right)^8$; d) $24(14x+4)\left(7x^2+4x+22\right)^3$;
e) $\frac{91}{3}(14x+9)\left(7x^2+9x+24\right)^{10/3}$; f) $22(14x+4)\left(7x^2+4x+29\right)^{19/3}$
g) $\frac{5(2x+8)}{2\sqrt{x^2+8x+25}}$; h) $\frac{7(14x+3)}{2\sqrt{7x^2+3x+24}}$;
i) $-\frac{4(6x+3)}{(3x^2+3x+22)^{3/2}}$; j) $-\frac{3(4x+2)}{2(2x^2+2x+22)^{3/2}}$;

Sine and Cosine - Chain Rules a,b are constants.

Function	Derivative	
$y = \sin(x)$	$\frac{dy}{dx} = \cos(x)$	Sine Rule
$y = \cos(x)$	$\frac{dy}{dx} = -\sin(x)$	Cosine Rule
$y = a \cdot \sin(u)$	$\frac{dy}{dx} = a \cdot \cos(u) \cdot \frac{du}{dx}$	**Chain-Sine Rule**
$y = a \cdot \cos(u)$	$\frac{dy}{dx} = -a \cdot \sin(u) \cdot \frac{du}{dx}$	**Chain-Cosine Rule**

Ex2a. Find $\frac{dy}{dx}$ where $y = 2\sin\left(9x^3 + 3x^2 + 1\right)$

Answer: $\mathbf{2\left(27x^2 + 6x\right)\cos\left(9x^3 + 3x^2 + 1\right)}$

$a = 2$

$u = 9x^3 + 3x^2 + 1 \quad \Rightarrow \frac{du}{dx} = 27x^2 + 6x$

Ex2b. Find $\frac{dy}{dx}$ where $y = 5\cos\left(9x^5 + 5x^4 + 3\right)$

Answer: $\mathbf{-5\left(45x^4 + 20x^3\right)\sin\left(9x^5 + 5x^4 + 3\right)}$

$a = 5$

$u = 9x^5 + 5x^4 + 3 \quad \Rightarrow \frac{du}{dx} = 45x^4 + 20x^3$

Ex2c. Find the derivative of $y = \left(4\sin(4x) + 4\cos(5x)\right)^4$

Answer: $\mathbf{4 \cdot \left(16\cos(4x) - 20\sin(5x)\right) \cdot \left(4\sin(4x) + 4\cos(5x)\right)^3}$

$n = 4$

$u = 4\sin(4x) + 4\cos(5x)$

$\Rightarrow \frac{du}{dx} = 16\cos(4x) - 20\sin(5x)$

Exercises

Find the derivatives of the expressions

a) $7\sin\left(7x^3+7x^2+8\right)$

b) $5\cos\left(2x^4+7x^3+9\right)$

c) $8\sin\left(8x^3+9x^2+1\right)$

d) $5\cos\left(6x^4+8x^3+2\right)$

e) $\left(\sin(x)+9\cos\left(x^4\right)\right)^4$

f) $\left(2\sin(6x)+9\cos\left(3x^2\right)\right)^3$

g) $\left(4\sin\left(5x^3\right)+8\cos\left(8x^5\right)\right)^5$

h) $\left(\sin\left(7x^3\right)+2\cos\left(6x^4\right)\right)^3$

i) $\left(5\sin\left(5x^5\right)+5\cos\left(7x^4\right)\right)^{\frac{1}{2}}$

j) $\left(6\sin\left(2x^3\right)+5\cos\left(8x^2\right)\right)^{\frac{3}{2}}$

Answers a) $7\left(21x^2+14x\right)\cos\left(7x^3+7x^2+8\right)$; b) $-5\left(8x^3+21x^2\right)\sin\left(2x^4+7x^3+9\right)$;

c) $8\left(24x^2+18x\right)\cos\left(8x^3+9x^2+1\right)$; d) $-5\left(24x^3+24x^2\right)\sin\left(6x^4+8x^3+2\right)$;

e) $4\cdot\left(\cos(x)-36x^3\sin\left(x^4\right)\right)\cdot\left(\sin(x)+9\cos\left(x^4\right)\right)^3$;

f) $3\cdot\left(12\cos(6x)-54x\sin\left(3x^2\right)\right)\cdot\left(2\sin(6x)+9\cos\left(3x^2\right)\right)^2$;

g) $5\cdot\left(60x^2\cos\left(5x^3\right)-320x^4\sin\left(8x^5\right)\right)\cdot\left(4\sin\left(5x^3\right)+8\cos\left(8x^5\right)\right)^4$;

h) $3\cdot\left(21x^2\cos\left(7x^3\right)-48x^3\sin\left(6x^4\right)\right)\cdot\left(\sin\left(7x^3\right)+2\cos\left(6x^4\right)\right)^2$;

i) $\frac{1}{2}\cdot\left(125x^4\cos\left(5x^5\right)-140x^3\sin\left(7x^4\right)\right)\cdot\left(5\sin\left(5x^5\right)+5\cos\left(7x^4\right)\right)^{-\frac{1}{2}}$;

j) $\frac{3}{2}\cdot\left(36x^2\cos\left(2x^3\right)-80x\sin\left(8x^2\right)\right)\cdot\left(6\sin\left(2x^3\right)+5\cos\left(8x^2\right)\right)^{\frac{1}{2}}$;

AP Calculus **Name: __________________________; Period: _____**

Derivatives with Charts!!

① *Working with Numerical Values* Suppose that a function f and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	−2
1	−3	1/5

Find the first derivative of the following combinations at the given value of x.

(a) $\sqrt{x}f(x)$, $x = 1$

(b) $\sqrt{f(x)}$, $x = 0$

(c) $f(\sqrt{x})$, $x = 1$

(d) $f(1 - 5\tan x)$, $x = 0$

(e) $\dfrac{f(x)}{2 + \cos x}$, $x = 0$

(f) $10 \sin\left(\dfrac{\pi x}{2}\right) f^2(x)$, $x = 1$

② *Working with Numerical Values* Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
−1	0	−1	2	1
0	−1	−3	−2	4

Find the first derivative of the following combinations at the given value of x.

(a) $3f(x) - g(x)$, $x = -1$

(b) $f^2(x)g^3(x)$, $x = 0$

(c) $g(f(x))$, $x = -1$

(d) $f(g(x))$, $x = -1$

(e) $\dfrac{f(x)}{g(x) + 2}$, $x = 0$

(f) $g(x + f(x))$, $x = 0$

③ Use the table to estimate the value of $h'(0.5)$, where $h(x) = f(g(x))$.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	12.6	14.8	18.4	23.0	25.9	27.5	29.1
$g(x)$	0.58	0.40	0.37	0.26	0.17	0.10	0.05

④ Given the following table of values at $x = 1$ and $x = -2$, find the indicated derivatives in parts (a)–(l).

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	−2	−1
−2	−2	−5	1	7

(a) $\dfrac{d}{dx}[f^2(x) - 3g(x^2)]\Big|_{x=1}$

(b) $\dfrac{d}{dx}[f(x)g(x)]\Big|_{x=1}$

(c) $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]\Big|_{x=-2}$

(d) $\dfrac{d}{dx}\left[\dfrac{g(x)}{f(x)}\right]\Big|_{x=-2}$

(e) $\dfrac{d}{dx}[f(g(x))]\Big|_{x=1}$

(f) $\dfrac{d}{dx}[f(g(x))]\Big|_{x=-2}$

(g) $\dfrac{d}{dx}[g(f(x))]\Big|_{x=-2}$

(h) $\dfrac{d}{dx}[g(g(x))]\Big|_{x=-2}$

(i) $\dfrac{d}{dx}[f(g(4 - 6x))]\Big|_{x=1}$

(j) $\dfrac{d}{dx}[g^3(x)]\Big|_{x=1}$

(k) $\dfrac{d}{dx}[\sqrt{f(x)}]\Big|_{x=1}$

(l) $\dfrac{d}{dx}[f(-\tfrac{1}{2}x)]\Big|_{x=-2}$

⑤ If $g(x) = f(f(x))$, use the table to estimate the value of $g'(1)$.

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.7	1.8	2.0	2.4	3.1	4.4

Chain Rule Project

Identify each of the following functions as a composition of functions. Then find its derivative.

1. $f(x) = (4x^3 - 2x^2 + x - 7)^3$
2. $g(x) = (x^4 + 5x^2 + 8)^{-4}$
3. $k(x) = \frac{x^2 - 5x + 3}{(x+1)^2}$
4. $f(r) = (r^3 - 4r + 1)^5$
5. $g(y) = \cos(2 + 5y)$
6. $h(t) = \sin^3(2t - 1)$
7. $G(u) = \sin(2u)\cos(3u)$
8. $H(t) = \sin^2 t + \sin t^2$
9. $J(x) = \left(\frac{3x^2 - 5}{2x^2 + 7}\right)^2$
10. $F(z) = \left(z^2 - \frac{1}{z^2}\right)$
11. $u(t) = t^2 \cos(1/t)$
12. $f(\theta) = (\theta^4 + cos^4\theta)^4$

Chain Rule with Trig. Function Practice (NO CALCULATOR)!!!

1. If $f(x)=x^3\cos x$, find $f'(\pi)$

2. If $f(x)=\sqrt{x}\sin x$, find $f'(2\pi)$

3. If $f(x)=\dfrac{\sec x}{x}$, find $f'(x)$

4. If $f(x)=\dfrac{\tan x}{x^2}$, find $f'(x)$

5. If $f(x)=\sin(2x)$, find $f'\left(\dfrac{\pi}{2}\right)$

6. If $f(x)=\cos\left(\dfrac{x}{2}\right)$, find $f'\left(\dfrac{\pi}{2}\right)$

7. If $f(x)=\sec(x^2)$, find $f'(\sqrt{\pi})$

8. If $f(x)=\tan^2 x$, find $f'\left(\dfrac{\pi}{4}\right)$

9. If $f(x)=\csc^5(x)$, find $f'\left(\dfrac{\pi}{2}\right)$

10. If $f(x)=\cos^3(4x)$, find $f'\left(\dfrac{\pi}{16}\right)$

11. If $f(x)=\cot^3(2x)$, find $f'\left(\dfrac{\pi}{12}\right)$

12. If $f(x)=\sin^3(3x^2)$, find $f'\left(\dfrac{\sqrt{\pi}}{3}\right)$

Equations of Tangent/Normal Lines Practice

1. Let f be the function defined by $f(x)=4x^3-5x+3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x=-1$?

A. $y=7x-3$
B. $y=7x+7$
C. $y=7x+11$
D. $y=-5x-1$
E. $y=-5x-5$

2. An equation of the line tangent to the graph of $y=x+\cos x$ at the point $(0,1)$ is

A. $y=2x+1$
B. $y=x+1$
C. $y=x$
D. $y=x-1$
E. $y=0$

3. The equation of the tangent line to the curve $y=\dfrac{3x+4}{4x-3}$ at the point $(1,7)$ is

A. $y+25x=32$
B. $y-31x=-24$
C. $y-7x=0$
D. $y+5x=12$
E. $y-25x=-18$

4. What is the equation of the line normal to the graph of $f(x)=x\sqrt{x^2+5}$ at $x=2$?

A. $y-6=\dfrac{3}{13}(x-2)$
B. $y-2=-\dfrac{3}{13}(x-6)$
C. $y-3=-\dfrac{3}{13}(x-2)$
D. $y-6=-\dfrac{3}{11}(x-2)$
E. $y-6=-\dfrac{3}{13}(x-2)$

5. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3,2)$?

A. 0
B. $\frac{4}{9}$
C. $\frac{7}{9}$
D. $\frac{6}{7}$
E. $\frac{5}{3}$

6. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

A. $y - 1 = -\left(x - \frac{\pi}{4}\right)$
B. $y - 1 = -2\left(x - \frac{\pi}{4}\right)$
C. $y = 2\left(x - \frac{\pi}{4}\right)$
D. $y = -\left(x - \frac{\pi}{4}\right)$
E. $y = -2\left(x - \frac{\pi}{4}\right)$

7. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5,-12)$ is

A. $5y - 12x = -120$
B. $5x - 12y = 119$
C. $5x - 12y = 169$
D. $12x + 5y = 0$
E. $12x + 5y = 169$

8. The equation of the normal line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

A. $y + 12x = 38$
B. $y - 4x = 10$
C. $y + 2x = 4$
D. $y + 2x = 8$
E. $y - 2x = -4$

9. The equation of the tangent line to the graph of $y = \cos x + \tan(2x)$ at the point $(0,1)$ is

A. $y = 0$
B. $y = 2x + 1$
C. $y = 2x$
D. $y = 2x - 1$
E. $y = x + 1$

10. If the line tangent to the graph of the function f at the point $(1,7)$ passes through the point $(-2,-2)$, then $f'(1)$ is

A. -5
B. 1
C. 3
D. 7
E. undefined

11. What is the equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

A. $y = 5x - 10$
B. $y = 3x + 4$
C. $y = 3x + 8$
D. $y = 3x - 10$
E. $y = 3x - 16$

12. The tangent line to the graph $y = e^{2-x}$ at the point $(1, e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

A. $2e$
B. $e^2 - 1$
C. e^2
D. $2e\sqrt{e}$
E. $4e$

12. The equation of the tangent line to the curve $y = x^3 - 6x^2$ at its point of inflection is

A. $y = -12x + 8$
B. $y = -12x + 40$
C. $y = 12x - 8$
D. $y = -12x + 12$
E. $y = 12x - 40$