

## Introduction

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Graphing calculators are powerful tools that we can use to quickly calculate numeric expressions, graph complicated functions, and analyze statistics.

This manual helps you use this tool effectively by showing you detailed keystroke directions that guide you through general directions and specific examples.

### Which Graphing Calculators

The keystrokes given in this manual work for the following calculators:

**TI-84 Plus**

**TI-84 Plus SE**

**TI-83 Plus**

**TI-83 Plus SE**

### Other Graphing Calculators

The examples in this manual will also generally work for the following calculators: (You may need to adjust a few keystrokes.)

**TI-83**

**TI-82**

Because mathematics is the same regardless of the calculator, the keystrokes given in this manual may also work for **other graphing calculators**.

### Algebraic Calculators

The keystrokes given in this manual will **not work** for **algebraic calculators**.

“Algebraic calculators” are calculators that do algebra, like the following:

**TI-89**

**TI-92**

If you have a TI-89, look for the “TI-89 Mathematics Manual.”

### How to Use This Manual

If you need help with a specific topic, look for it in one of the following:

1. The Brief Table of Contents – an overview of topics in the order they appear
2. The Extended Table of Contents – a detailed list of topics in the order they appear
3. The Index – an alphabetical list of topics

You can also learn how to use many features of your calculator by starting on the first page of this manual and continuing to the end. Read the general directions. Then work the examples, entering the keystrokes given on your calculator.



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## TI-84/83 Mathematics Manual

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# 1 Calculations With a Graphing Calculator

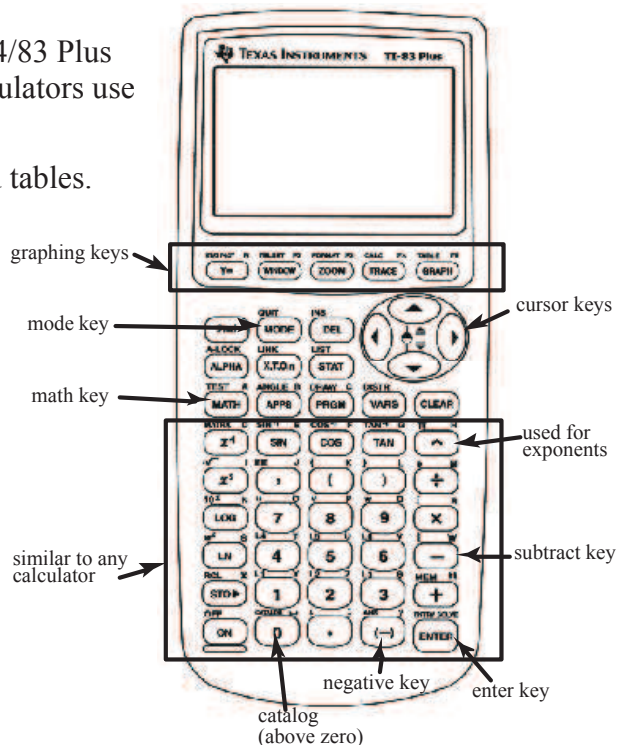
## Layout

This diagram shows the layout of a TI-84/83 Plus graphing calculator. Most graphing calculators use a similar layout.

Use keys in the top row for graphing and tables.

Advanced functions, editing keys, and the cursor keys are in the next three rows. Use the cursor keys to move the cursor around the calculator's screen.

The bottom section of your graphing calculator is similar to any scientific calculator. It includes the numbers, mathematical operations, and the trigonometric and exponential functions.



## The **Second** and **Alpha** Keys

Each key is three "keys" in one. It's **color-coded**. Press the second key to select a command in that color. Press the alpha key before pressing letters.

## Screen Contrast

Adjust your screen contrast so you can read it by pressing **ON** and then

**2<sup>nd</sup>** **▲** [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] to make it **darker**,

or **2<sup>nd</sup>** **▼** [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] to make it **lighter**.

## The Cursor

The cursor is the square, blinking rectangle on the home screen.

When you press **ALPHA** the rectangle shows the letter "A".

When you press **2<sup>nd</sup>** the rectangle shows an up arrow.

When you press **2<sup>nd</sup>** **INS** the rectangle becomes a blinking underline.

**Menu Keys**

Some keys, when pressed, give you a menu of choices. You can get the option you want by moving to that option with the cursor keys and pressing **ENTER**. Or you can press the number of the option you want.

**Important Keys**

<b>ENTER</b>	<b>Enter key</b> – in the lower right corner, press to do the calculation
<b>-</b>	<b>Subtract or minus key</b> – in the right column of your keyboard
<b>(-)</b>	<b>Negative key</b> – in the bottom row of your keyboard, next to <b>ENTER</b>
<b>^</b>	<b>Raised to</b> – the caret symbol in the right column, used for exponents
<b>√</b>	<b>Square root</b> – above the $x^2$ key, press <b>2<sup>nd</sup></b> <b>√</b>
<b>MATH</b>	<b>Math</b> – a menu of several mathematical operations in the left column, below the alpha key
<b><math>\sqrt[x]{\phantom{x}}</math></b>	<b>xth root</b> – press the index needed, then <b>MATH</b> , and option 5
<b>►Frac</b>	<b>Fraction form</b> – converts the answer to a fraction form option 1 under <b>MATH</b>
<b>EE</b>	<b>Scientific notation exponent</b> -- symbol used to enter powers of 10 above the comma key
<b>MODE</b>	<b>Mode</b> – used to change the calculator's mode to the right of the second key
<b>QUIT</b>	<b>Quit</b> – press <b>2<sup>nd</sup></b> <b>QUIT</b> ( <b>MODE</b> ) to leave a menu
<b>DEL</b>	<b>Delete key</b> – move the cursor on top of what you want to delete, press <b>DEL</b> to the right of <b>MODE</b>

**INS**      **Insert** – move the cursor to the desired location and press  $\boxed{2^{\text{nd}}}$  **INS**  
 (  $\boxed{\square\square\square\square}\boxed{\text{DEL}}$  ), the cursor becomes a blinking underline

**ENTRY**    **Entry** – above  $\boxed{\text{ENTER}}$ , press  $\boxed{2^{\text{nd}}}$  **ENTRY**  
 used to return previous expressions so they can be edited

**ANS**      **Answer** – above  $\boxed{(-)}$ , press  $\boxed{2^{\text{nd}}}$  **ANS**  
 used to return the most recent answer

$\boxed{\text{STO}\blacktriangleright}$     **Store** – the key above the  $\boxed{\text{ON}}$  key  
 used to store a value in memory  
 type the value you wish to store, then press  $\boxed{\text{STO}\blacktriangleright}$   $\boxed{\text{ALPHA}}$   
 press any alphabetic letter and then press  $\boxed{\text{ENTER}}$   
 only single letter variables can be used  
 pressing  $\boxed{\text{STO}\blacktriangleright}$  **before** typing a value stores the previous answer

### The MATH Menu

The MATH menu lists several mathematical operations.

Press  $\boxed{\text{MATH}}$  to see the math menu.

Several options are available, including Option 1  
 which converts answers to fraction form.  
 Option 4 is the cube root, while option 5 is the  $x$ th root.  
 The arrow at the bottom of the screen shows you that you  
 can find more items in the menu by pressing the down arrow.



There are four menus available.

Press the right cursor arrow to see the choices available in the number (NUM)  
 menu.

Explore the complex and probability menus on your own.

To select a menu item, highlight it using the cursor arrows  
 and press  $\boxed{\text{ENTER}}$ . You can also type in the option number.

If you do not select an option, press  $\boxed{2^{\text{nd}}}$  **QUIT** to return to the  
 home screen.











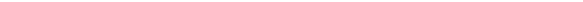
## Storing a Number

**Example 8** Store 3.1 in memory  $\square$  and find (a)  $3\square^2$  and (b)  $(3\square^2)^{10}$

[illegible]

(a) 

$$3 \square^2 = 28.33$$

(b) 

$$(3\Box^2)^{10} \approx 3.966855894 \times 10^{14}.$$

When an answer exceeds the screen capacity it is given in scientific notation.

## Entering Numbers in Scientific Notation

**Example 9** Find the product of  $7.34 \times 10^7$  and  $2.17 \times 10^{-3}$ .

[illegible]

7 x 2 . 1 7 2<sup>nd</sup> EE (-) 3 ENTER

MODE.

☐ ☐☐☐ ☐☐☐ ☐☐☐☐ ☐☐ SCI and press .

Press **2<sup>nd</sup>** QUIT to return to the home screen.

Then press **ENTER** to get the product in scientific notation.

Rounding to three significant digits, we have the product  $1.59 \times 10^5$

NOTE: Your calculator stays in scientific notation until you change it back to normal mode.

To do this, press **MODE**, press **ENTER** to select NORMAL and **2<sup>nd</sup>** QUIT to return to the home screen.

Complex/Imaginary Numbers

**Example 10** Find the difference, product and quotient of  $2 + 3i$  and  $5 - 7i$ .

First put your calculator into complex/imaginary mode.

To do this press **MODE**,

move the cursor down to Real and across to a + bi

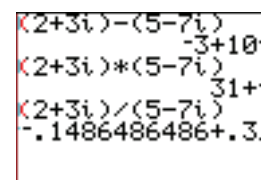
then press **ENTER** to select it

and **2<sup>nd</sup>** QUIT to return to the home screen.



Type each number grouped in parentheses.

Press **2<sup>nd</sup>** i to move the decimal point to type in the i.



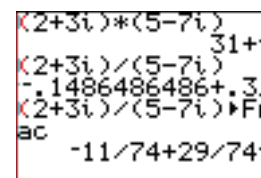
Type in the difference first and press **ENTER**,

Then press **2<sup>nd</sup>** ENTRY and edit the operation to

quickly find the product and the quotient.

If you would like the quotient in fraction form,

Press **2<sup>nd</sup>** ENTRY **MATH** 1 **ENTER**.



NOTE: Your calculator stays in complex notation until you change it back to real mode.

To do this, press **MODE**, move the cursor to Real and

press **ENTER** and **2<sup>nd</sup>** QUIT to return to the home screen.

## 2 Graphing Functions with a Calculator

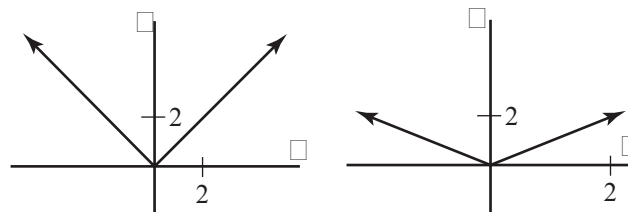
### The Graphing Calculator's Technique

**Graphing calculators graph using the “table-of-values” or “point-plotting” method.** The  $x$ -axis is divided into intervals, depending on the number of pixels across the screen, the window used, and the  $x$ -resolution setting. Each  $x$ -value is put into the function. Then  $y$ -values are calculated. The calculator plots these ordered pairs. Then it connects them.

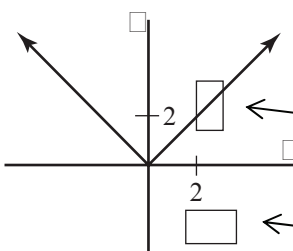
### The Scale

Here are two graphs of  $y = |x|$ .

They look different because different scales were used for the  $x$ -axis. When you use a calculator to graph, you need to tell the calculator what scale to use.



### The Viewing Window



The section visible in the calculator's screen is called the **viewing window**.

If you used this viewing window, then you might think that you had a line.

If you used this one, then you might think there is no graph.

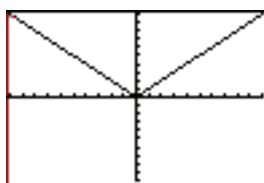
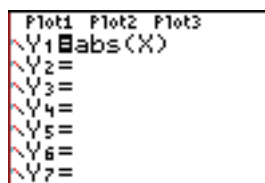
Carefully consider the viewing window you need for the function you are graphing.

**Example 1** Graph  $y = |x|$  using your graphing calculator.

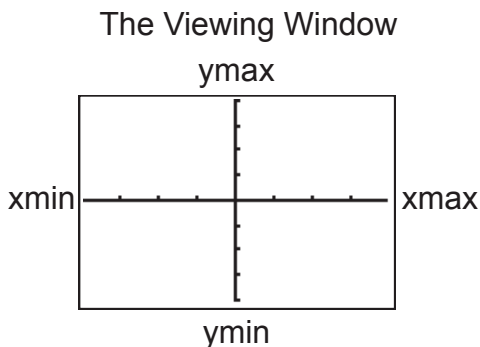
Press  $y=$  **MATH** move to the right to select NUM  
and press **ENTER** to select ABS for an absolute value.

Then press  $x$  **)**. The  $x$  is below the **MODE**  $e$ .

Then press **GRAPH** to see the graph.



What you see on **your**  
calculator will depend  
upon **your** viewing window.



Press **WINDOW** to see **your**  
window values.

The Xmin and Xmax shown are the x-values at the edge of the window.  
The Ymin and Ymax shown are the y-values at the edge of the window.  
The Xscl and Yscl listed in WINDOW are the “scale values” – the value of the distance between each tick mark on the x- or y-axis.

### Adjusting the Window

There are two ways to change your window: using the window screen or using the zoom options. There are several zoom options, all listed below. Try a few using your graph of the absolute value function from Example 1, and then skip ahead to the next examples. You can refer back to this complete list as needed.

Using WINDOW

Press **WINDOW** and type in the changes. Press **GRAPH** to see how this changes the window. See Example 1 for a graphic that shows the window values.

Using ZOOM

Press **ZOOM**

to see the built-in viewing windows.

Notice the arrow next to the 7.

Use the cursor down arrow to see options 8, 9 and 0.

```

ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig

```

```

ZOOM MEMORY
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9:ZoomStat
0:ZoomFit

```

ZBox

The first option, **zoom-box**, is used to “box in” a section of the graph that you find interesting.

Select it by pressing **ZOOM** **1**

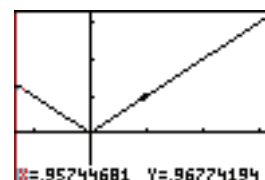
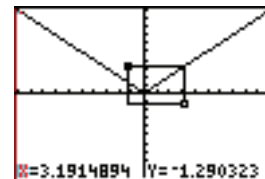
Or press **ZOOM** **ENTER**.

Then use the cursor arrows to move to one corner of your box.

Press **ENTER**.

Now move to the diagonally opposite corner.

Press **ENTER** again and the window will adjust to the boxed region.

ZOOM In/Out

**Zoom-in** and **zoom-out**, work like a video camera – zooming in or out with the window centered at the cursor location. Select the option.

Move the cursor to the desired center and press **ENTER**.

ZDecimal

The **zoom-decimal** window has a change in  $x$  of one-tenth, so coordinates on your curve will be in multiples of 0.1.

Press **ZOOM** **4** to select the zoom-decimal window.

You can find larger or smaller “friendly” windows by multiplying the zoom-decimal window values by any constant.

```

WINDOW
Xmin=-4.7
Xmax=4.7
Xscl=1
Ymin=-3.1
Ymax=3.1
Yscl=1
Xres=1

```

ZSquare

The **zoom-square** window adjusts the window so that the tick marks on both axes are the same width apart.

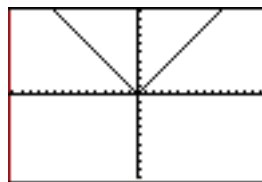
Values will depend upon the previous window used.

Press **ZOOM** **5** to select a square window.

```

WINDOW
Xmin=-15.16129...
Xmax=15.161290...
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```

ZStandard

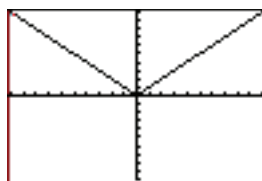
A **standard window** shows both  $x$ - and  $y$ -values from  $-10$  to  $10$ . Each tick mark on the axes stands for one unit. Notice that the shape of your calculator's screen is wider than it is high. So one unit on the  $x$ -axis is wider than one unit on the  $y$ -axis.

Press **ZOOM** **6** to select a standard window.

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```

ZTrig

The **zoom-trigonometry** window is designed to show the standard trigonometric functions. See Section 3 Trigonometry for more information about this window.

ZInteger

The **zoom-integer** window gives integer values for the coordinates on the curve.

Press **ZOOM** **8** and then **ENTER** to select the center of the integer window.

```

WINDOW
Xmin=-47
Xmax=47
Xscl=10
Ymin=-31
Ymax=31
Yscl=10
Xres=1

```

ZoomStat

The **zoom-statistics** window sets the viewing window to include all statistical data points from a table you have created.  
See Section 9 Probability and Statistics for more information about this window.

ZoomFit

The **zoom-fit** window recalculates the Ymin and Ymax using the given  $x$ -values for the window. Only the  $y$ -values for the window change. The  $x$ -values remain the same.

Press **[ZOOM]** **[0]** to “fit” your curve to the given  $x$ -values.

**WARNING:** Use the zoom-fit window carefully, remembering that it does not adjust the  $x$ -values. You may need to adjust your  $x$ -values to show important parts of your curve.

Zoom Memory

Press **[ZOOM]** and move the cursor to the right to MEMORY to see the zoom memory options.

ZPrevious

Option 1 returns the graphing screen to the graphing window immediately preceding the current graphing window.

ZoomSto/ZoomRcl

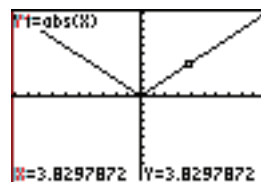
Options 2 and 3 allow you to store (ZoomSto) or recall (ZoomRcl) a graphing window.

The Trace Command

The trace command allows you to move the cursor along the curve.  
You can also see the coordinates of the ordered pairs that were used to graph the curve.

Press **[TRACE]**.

Use the right and left cursor arrows to move along the curve.  
Use the up and down cursor arrows to move between curves.







Let's improve it using what we know about it in

as in we let  $x^3 - 100x = x(x - 10)(x + 10)$ .

to our favorite losses the x-axis at the origin and

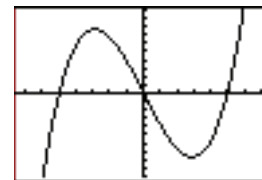
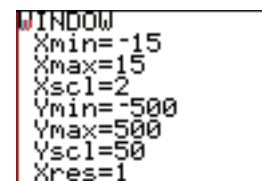
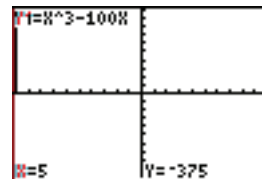
use an Xmin of and Xmax of with an x-scale of in the.

Press **TRACE**, then **5** **ENTER**, to see that the y-value is when x is

Let's use a Ymin of and Ymax of with a y-scale of in the include -375 and 0.

Press **WINDOW** and change the window values to the new values.

Press **ENTER** at the end of one. Then press **GRAPH**.



Now we have a better view of our curve.

There are many other appropriate windows.

A good window shows the

x- and y-intercepts and

the general shape of the curve,

or the region of the curve that you wish to see.

**Example 3** Graph using your calculator.  $f(x) = x^3 - 0.01x$

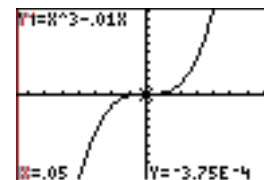
divide y = screen to the new union.

as in  $x^3 - 0.01x = x(x - 0.1)(x + 0.1)$ .

to the x-intercepts at the origin and

Press **TRACE** and **0** **.** **0** **5** **ENTER**

an x-value between 0 and



Then we have a y-value of  $-3.75 \times 10^{-4} = -0.000375$ . This y-value is very close to 0.

Change the window to the

window shown on the right

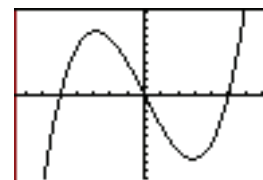
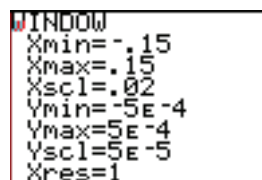
and press **GRAPH**.

Let's see what means

$5 \times 10^{-4} = 0.0005$ .

Why is this less than

then **2<sup>nd</sup>** **EE** above the decimal key and then

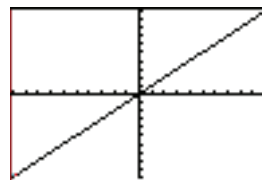
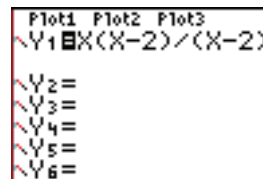


**Example 4** Graph using your calculator.  $f(x) = \frac{x(x-2)}{(x-2)} = \begin{cases} x & \text{if } x \neq 2 \\ \text{undefined} & \text{if } x = 2 \end{cases}$

Press  $y=$   $x$   $($   $x$   $-$   $2$   $)$   $\div$   $($   $x$   $-$   $2$   $)$   $\text{ZOOM}$   $6$

This should be the line  $y = x$  with a hole at  $x = 2$ .

But the hole does not show in a standard window.  
This is because the calculator never evaluated the function at  $x = 2$ .



Press  $\text{TRACE}$  and move the cursor so see  
the actual points plotted and connected.

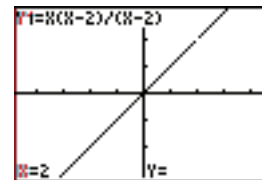
Your calculator **will** evaluate the function  
at  $x$  equals 2 in a **zoom-decimal window**.

Press  $\text{ZOOM}$   $4$

And now we can see the hole in the line.

Press  $\text{TRACE}$  and  $2$   $\text{ENTER}$  to

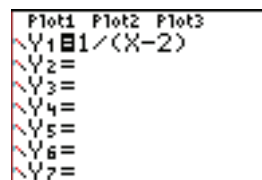
see that there is no  $y$ -value at



**Example 5** Graph using your calculator.  $f(x) = \frac{1}{x-2}$

Notice that  $y =$  screen to see this new function.

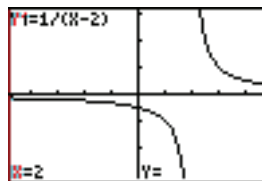
Press  $\text{GRAPH}$  to see the graph of the function  
in a zoom-decimal window.



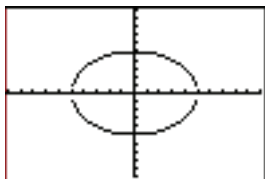
Notice that the calculator does **not** draw asymptotes.

Press  $\text{TRACE}$  and  $2$   $\text{ENTER}$  to see that there is

no  $y$ -value at



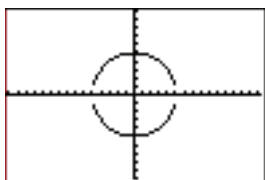


zoom standard

In this window, the circle looks like an ellipse.

The tick marks that stand for one unit are further apart on the  $x$ -axis than the  $y$ -axis.

Use a zoom-square window to get a scale that has the same distance between the tick marks on each axis.

zoom-square

Press **ZOOM** **5**

Now we have a “round” circle.

The two halves are not connected.

Each half of the circle was graphed using a different function.

So the calculator does not know to connect them. But we can interpret this as a completed circle from what we know about the equation.

Graphing Piecewise Functions

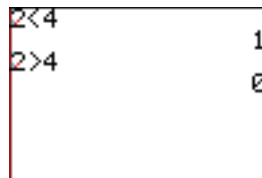
We can graph piecewise functions on a graphing calculator using inequalities.

When you enter an inequality on your calculator,

it is tested to see if it is true or false.

Your calculator gives a value of one if the inequality is true.

It gives a value of zero if the inequality is false.



To find the inequality symbols press **2<sup>nd</sup>** TEST (above **MATH**)

If you **divide** a function's formula by an inequality, your calculator graphs points only where the inequality is true.

$$\text{So } \square(x) = x^2 \div (x \geq 2) = \begin{cases} x^2 & \text{if } x \geq 2 \\ \text{undefined} & \text{if } x < 2 \end{cases}$$

Your calculator graphs only the part of the parabola where  $x$  is larger than or equal to two. Do this separately for each piece of a piecewise function to get a good graph of the function.

**Example 8** Graph  $f(x) = \begin{cases} x^2 & \text{if } x \geq 2 \\ x & \text{if } x < 2 \end{cases}$  on your graphing calculator.

*Step 1: Enter the function into the calculator.*

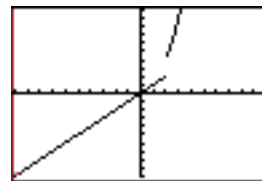
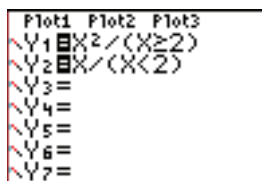
Press  $y =$   $x$   $x^2$   $\div$   $($   $x$   $2^{nd}$  Test  $\text{above}$   $\text{MATH}$   $)$   $4$   $\text{above}$   $\text{MATH}$   $)$   $2$   $)$ .

Now press  $\text{ENTER}$   $\text{above}$   $y_2$ .

and  $x$   $\div$   $($   $x$   $2^{nd}$  Test  $5$   $\text{above}$   $\text{MATH}$   $)$   $2$   $)$ .

Press  $\text{ZOOM}$   $6$  to see the graph in a standard window.

Although it looks like a line, the curve on the right is the part of  $y = x^2$  where  $x$  is larger than or equal to two.



**WARNING:** When you sketch this on paper, remember to carefully mark whether each endpoint is an open endpoint or a closed endpoint.

### Evaluation

We can use a calculator to find values of a function using

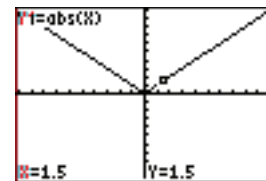
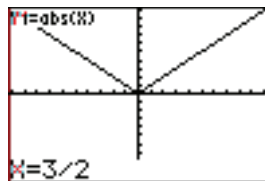
1. the TRACE command
2. using function notation
- and 3. using a table.

### Using the Trace Command to Find Function Values

You can use the trace command to find function values.

Press  $\text{TRACE}$ .

Type an  $x$ -value and press  $\text{ENTER}$  to find a function value at a specific  $x$ -value in the window.



WARNING: If the  $x$ -value is outside of the viewing window, then you will get an error message.

Press **ENTER**. Then adjust the Xmin or Xmax in your viewing window to include the desired  $x$ -value. Then use the trace command again.



### Function Notation

Before you can use function notation, the function must be entered in the  $y=$  screen. Then you can use function notation in the home screen to evaluate a function.

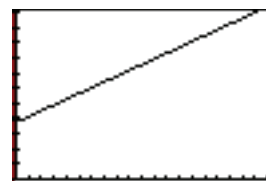
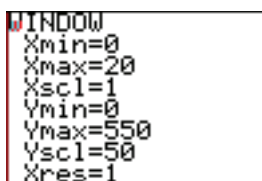
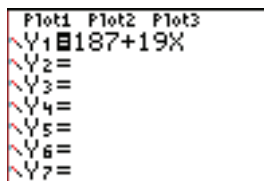
Press **VAR**, select Y-VARS, select Option 1 (Function) and then pick your function.

**Example 9** Assume that the fine for speeding on a certain freeway is \$187 plus \$19 for every mile per hour  $x$  over 65 mph. Use your calculator to graph a function representing the total fine for speeding. (a) Use trace to find the fine if someone is going 81 mph. (b) Use function notation to find the fine for going 100 mph.

(a)  $f(x) = 187 + 19x$ .

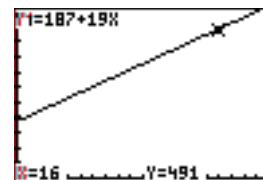
So someone is going  $x = 81 - 65 = 16$ .

Your window needs  $x$ -values at least as large as 0 and  $y$ -values large enough



Now press **TRACE** **1** **6** **ENTER**.

The fine for going 81 mph on a freeway with a speed limit of 65 mph is \$491.





ess 2<sup>nd</sup> TBLSET o a le se u

*a* ove WINDOW .

*ess*  4  ENTER  1.

```
TABLE SETUP
TblStart=4
ΔTbl=1
Indent: Auto Ask
Depend: Auto Ask
```

X	Y1
5	263
5	282
6	301
7	320
8	339
9	358
10	377

$\bar{x} = 4$

□o see □□e □le□

ess 2<sup>nd</sup> TABLE above GRAPH)

You can use the up and down cursor arrows to scroll through any table.

You can use the Ask feature to find values in our table.

In the table setup screen, change Auto to Ask.

### The $y$ -Intercept

Find the  $y$ -intercept by evaluating the function at zero, using TRACE, function notation or a table.

### $x$ -Intercepts

The places where the curve crosses the  $x$ -axis are called ZEROS on the TI-84/3 Plus and many other calculators.

To find  $x$ -intercepts (or zeros) press  $\boxed{2^{nd}}$  CALC (above  $\boxed{TRACE}$ ).

Select Option 2 for zero.

Select an  $x$ -value to the **left** of the desired zero by moving to it using the cursor arrows or typing it in and press **ENTER**. Then select an  $x$ -value to the **right** of the zero and press **ENTER**.

Finally, select a value close to the zero for your "guess" and press **ENTER**.

WARNING: Always be sure to press the third **ENTER** when you are finding a zero. The calculator screen should say "zero" above the resulting  $x$ -value.

WARNING: A function of  $x$  can have one, many or no  $x$ -intercepts. Be sure that you use a viewing window that shows **all** of the function's  $x$ -intercepts.

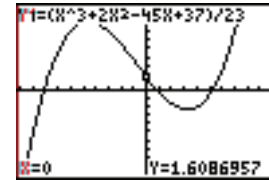


**Example 11** Find all axis intercepts for  $f(x) = \frac{x^3 + 2x^2 - 45x + 37}{23}$ .

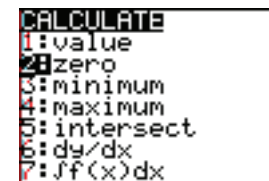
Use the union in  $\cap$  the  $y$  screen and  $\cap$   $\cap$  in a standard window.

To find the  $y$ -intercept press **TRACE** **0** **ENTER**.

The  $y$ -intercept is approximately 1.6.

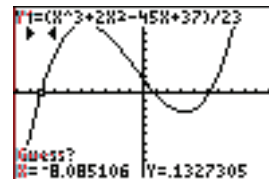


To find  $x$ -intercepts press **2<sup>nd</sup>** **CALC** **0** **ENTER** above **TRACE** **2** **ENTER**.

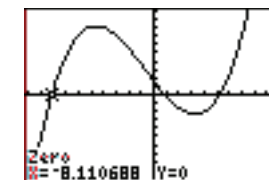


The calculator asks for a left bound. Move the cursor to the left so that it is to the left of the  $x$ -intercept we want. Press **ENTER**.

The calculator asks for a right bound. Move the cursor to the right so that it is to the right of the  $x$ -intercept we want. Press **ENTER**.



Notice the two  $x$ -values are on the screen. The calculator uses a numerical approximation method to find where the curve crosses the  $x$ -axis between those two  $x$ -values. When you are asked to guess move as close as you can to the  $x$ -intercept and press **ENTER**.



Notice the word “zero” is now on the screen. The  $x$ -value of the  $x$ -intercept is given.

The  $y$ -value should be either  $0$  or very close to  $0$ . The  $y$ -value is an **approximate value**.

The  $x$ -value is also  $0$  or very close to  $0$  or  $0$  or  $0$ . You wish you can also **type** values to the left bound and right bound and you guess.

The three  $x$ -intercepts for this function are approximately  $-8.11$ ,  $0.87$  and  $5.24$ .

Extrema

Local or relative extrema are the local “peaks” and “valleys” of a function’s graph. These can be found **exactly** using calculus. Your calculator can also find them **approximately** using numeric approximation techniques. The procedure for finding a local maximum or minimum is very similar to the procedure for finding an  $x$ -intercept.

Press  $2^{\text{nd}}$  CALC (above TRACE). Select  $\boxed{3}$  or  $\boxed{4}$  (for minimum or maximum). Select an  $x$ -value to the **left** of the desired minimum or maximum by moving to it using the cursor arrows or typing it in and press ENTER.

Then select an  $x$ -value to the **right** of the minimum or maximum and press ENTER. Finally, select a value close to the minimum or maximum for your “guess” and press ENTER.

WARNING: Always be sure to press the third ENTER when you are finding a minimum or maximum. The calculator screen should say “minimum” or “maximum” above the result.

**Example 12** Find the local minimum and maximum for  $f(x) = \frac{x^3 + 2x^2 - 45x + 37}{23}$ .

The slope of the function is entered in the  $y$ -screen and the local minimum or valley is visible in your graph window.

Press  $2^{\text{nd}}$  CALC to calculate  $\boxed{3}$  to find a minimum.

The calculator asks for a left bound.

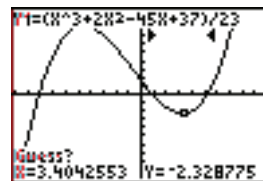
Over the cursor so that it is under the leftmost local

minimum. Press ENTER.

The calculator asks for a right bound.

Over the cursor so the cursor is under the

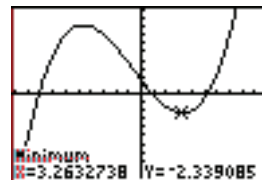
rightmost local minimum. Press ENTER.



Once the two points are on the screen below, the calculator uses a numeric maximization method to find the  $y$ -value between those two  $x$ -values.

Then you are asked to guess how close as you can to the local minimum and press **ENTER**.

Once the message “minimum” is now on the screen, the  $x$ -value is the approximate location of the local minimum. The  $y$ -value is approximately the smallest  $x$ -value in the neighborhood of the  $y$ -value.



The same is done using “maximum” to find the local maximum.

The **local minimum** for this function is approximately  $-2.34$  when  $x$  is about  $3.26$ . The **local maximum** for this function is approximately  $8.22$  when  $x$  is about  $-4.60$ .

### Graphing Inverse Relations

Your graphing calculator draws a function’s **inverse relation** by calculating ordered pairs for the function, reversing their order, and plotting these new points. If the original function is one-to-one, then the inverse relation is also an **inverse function**.

To draw the inverse function/relation using your calculator, first graph the function.

Then press **2<sup>nd</sup>** DRAW (above **PRGM**).

Select Option **8** DrawInv (for draw the inverse relation).

When this appears on the home screen, select the function that you want using **VARS** and press **ENTER**.

**Example 13** Draw the inverse relation for  $f(x) = \frac{x^3 + 2x^2 - 45x + 37}{23}$ .

Once the message is on the screen, we are asked to draw an inverse relation **not** an inverse function.

The same message is entered in the  $y$ -screen and you have a good view in the window.

Press **2<sup>nd</sup>** DRAW above **PRGM**.

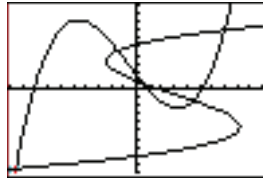
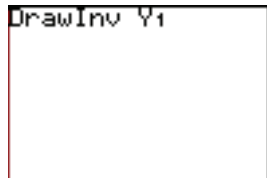
Select Option **8** DrawInv (to draw the inverse relation). DrawInv appears on the  $y$ -screen.

Then press **VARS** to variables and move to the **i** to select **Y-VARS**.

Press **ENTER** to select **Option 1** **union** and **ENTER** to select **Option 1**  $y_1$ .

You are now at the box seen.

Press **ENTER** on the box and the calculator draws the inverse relation.



To leave the inverse relation press **2<sup>nd</sup>** **DRAW** **Option 1** **ClrDraw** to leave the drawing.

This redraws the union without the inverse relation.

**WARNING:** You cannot use this inverse drawing to find values on the inverse relation.

The **TRACE** key does not work when an inverse curve is drawn this way.

### 3 Trigonometry

#### Radian/ Degree Mode

Before you can use your calculator to find trigonometric values or graphs, you must decide whether you are using degrees or radian measures. Then you need to set your calculator to the correct mode.

Follow these steps to do select degree or radian mode:

Press **MODE**.

Move down and across to Degree or Radian, and press **ENTER** to select it.

Press **2<sup>nd</sup>** **QUIT** to return to the home screen.

**WARNING:** Always check that you are using the mode that you intend by pressing **MODE** and checking which is highlighted.

#### Radian Mode



#### Degree Mode



#### Converting Radians to Degrees

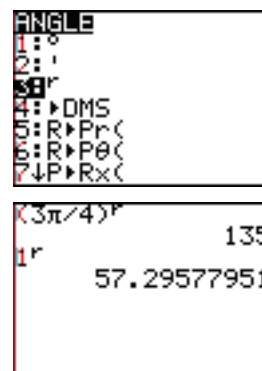
You must have your calculator in **degree mode** to convert a radian measure to degrees.

**Example 1** Convert from radians to degrees. a)  $\frac{3\pi}{4}$  b) 1

**First be sure that your calculator is in degree mode.**

- a) Press **(** **3** **2<sup>nd</sup>** **π** **÷** **4** **)**  
**2<sup>nd</sup>** **ANGLE** (above the **APPS** key)  
**3** (for radian) **ENTER**.  
 $\frac{3\pi}{4} = 135^\circ$

- b) Press **1** **2<sup>nd</sup>** **ANGLE** **3** (for radian) **ENTER**.  
 1 radian  $\approx 57.3^\circ$



### Converting Degrees to Radians

You must have your calculator in radian mode to convert a degree measure to radians.

**Example** □ Convert from degrees to radians. a)  $74^\circ$  b)  $45^\circ$

**First be sure that your calculator is in radian mode.**

- a) Press  $\boxed{7} \boxed{4} \boxed{2^{\text{nd}}} \text{ANGLE}$  (above the  $\boxed{\text{APPS}}$  key)  
 $\boxed{1}$  (for de $\square$ ree)  $\boxed{\text{ENTER}}$ .

$$74^\circ \approx 1.2915$$

- b) □ Repeat this procedure for  $45^\circ$ .

$$45^\circ \approx 0.7854$$

□ Did you recognize this as an approximation to  $\frac{\pi}{4}$  □

ANGLE	
1:	$^\circ$
2:	'
3:	"
4:	$\rightarrow$ DMS
5:	R $\rightarrow$ Prc
6:	R $\rightarrow$ P $\theta$
7:	P $\rightarrow$ R $\times$

$74^\circ$	1.291543646
$45^\circ$	.7853981634
$\pi/4$	.7853981634

### Approximate Values

**WARNING:** Remember that your calculator gives **approximate** (not exact) values when finding trigonometric values.

For example, although we know

that  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ , your calculator gives the

approximate result  $\cos \frac{\pi}{6} = 0.8660254038$ .

$\cos(\pi/6)$
.8660254038

### Finding Trigonometric Values Using Degree Mode

First set your calculator to degree mode.

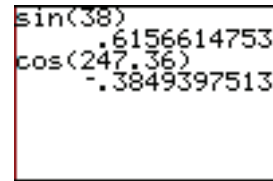
With your calculator, you can find approximate trigonometric values for any angle.

NORMAL	SCI	ENG
Float	0	1 2 3 4 5 6 7 8 9
RADIAN	DEGREE	
FUNC	PAR	POL SEQ
CONNECTED	DOT	
SEQUENTIAL	SIMUL	
REAL	a+bi	re $\theta$ i
FULL	HORIZ	G-T
SET CLOCK 07/31/09 1:55PM		

**Example** □ Use your calculator to approximate the following.

- a)  $\sin 38^\circ$  b)  $\cos 247.36^\circ$  c)  $\tan(-2103^\circ)$  d)  $\tan 90^\circ$

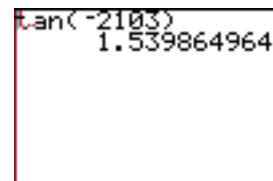
- a) Press  $\boxed{\text{SIN}} \boxed{3} \boxed{8} \boxed{)} \boxed{\text{ENTER}}$ .  
 $\sin 38^\circ \approx 0.6157$



- b) Press  $\boxed{\text{COS}} \boxed{2} \boxed{4} \boxed{7} \boxed{.} \boxed{3} \boxed{6} \boxed{)} \boxed{\text{ENTER}}$ .  
 $\cos 247.36^\circ \approx -0.3849$

□ Notice that your calculator can evaluate trigonometric functions at decimal degrees.

- c) Press  $\boxed{\text{TAN}} \boxed{(-)} \boxed{2} \boxed{1} \boxed{0} \boxed{3} \boxed{)} \boxed{\text{ENTER}}$  (negative sign to the  $\boxed{\text{ENTER}}$  key)



□ Notice that your calculator can evaluate trigonometric functions at negative angles.

$$\tan(-2103^\circ) \approx 1.5399$$

**WARNING:** Be sure to use the **negative** key.  
 Do not use the subtract key for a negative.

- d) Press  $\boxed{\text{TAN}} \boxed{9} \boxed{0} \boxed{)} \boxed{\text{ENTER}}$ .  
 □ Your calculator gives you an error message because the value of this quadrantal angle is undefined.



Press  $\boxed{\text{ENTER}}$  to return to the home screen.

$\tan 90^\circ$  is undefined.

### Finding Values for Secant, Cosecant and Cotangent Using Degree Mode

You may have noticed that your calculator does not have keys for secant, cosecant and cotangent.

Use the reciprocal identities to find values for secant, cosecant and cotangent.

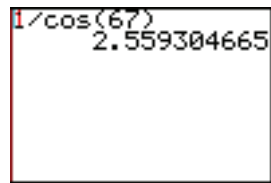
**Example** □ Use your calculator to approximate the following.

a)  $\sec 67^\circ$    b)  $\csc(-5342.7^\circ)$    c)  $\cot(27^\circ + 94^\circ)$    d)  $\cot 27^\circ + \cot 94^\circ$

a)  $\sec 67^\circ = \frac{1}{\cos 67^\circ}$

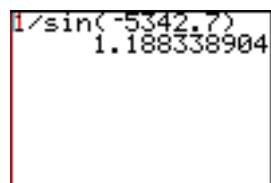
Press  $\boxed{1} \boxed{\div} \boxed{\cos} \boxed{6} \boxed{7} \boxed{)} \boxed{\text{ENTER}}$ .

$\sec 67^\circ \approx 2.5593$



b)  $\csc(-5342.7^\circ) = \frac{1}{\sin(-5342.7^\circ)}$

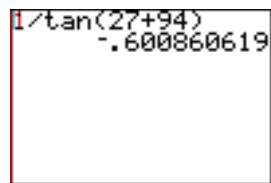
$\csc(-5342.7^\circ) \approx 1.1883$



□ In parts (c) and (d) □ be careful with the order of operations.

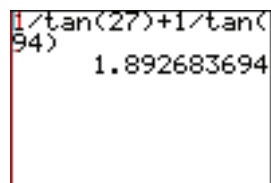
c)  $\cot(27^\circ + 94^\circ) = \frac{1}{\tan(27^\circ + 94^\circ)}$

$\cot(27^\circ + 94^\circ) \approx -0.6009$



d)  $\cot 27^\circ + \cot 94^\circ = \frac{1}{\tan 27^\circ} + \frac{1}{\tan 94^\circ}$

$\cot 27^\circ + \cot 94^\circ \approx 1.8927$



### Degrees, Minutes, Seconds

You can use your calculator to find trigonometric values when the angle is given in degrees, minutes and seconds.

**Example** □ Use your calculator to approximate  $\sin(12^\circ 17' 47'')$ .

Press  $\boxed{\sin} \boxed{1} \boxed{2} \boxed{2^{\text{nd}}}$  ANGLE (above  $\boxed{\text{APPS}}$ )  $\boxed{1}$  (for de□rees)  
 $\boxed{1} \boxed{7} \boxed{2^{\text{nd}}}$  ANGLE  $\boxed{2}$  (for minutes)  $\boxed{4} \boxed{7} \boxed{\text{ALPHA}}$  " (for seconds □  
 above the plus key)  $\boxed{)} \boxed{\text{ENTER}}$ .





$$\sin(12^\circ 17' 47'') \approx 0.2130$$

### Finding Acute Angles in Degrees

You can use your calculator to approximate an acute angle  $\theta$ , given a trigonometric value.

Remember that an acute angle is an angle between  $0^\circ$  and  $90^\circ$ .

**Example** □ Find an approximate **acute angle**  $\theta$  in degrees, given the following:

a)  $\sin \theta = 0.5432$     b)  $\tan \theta = \frac{\sqrt{3}}{3}$     c)  $\csc \theta = 3.4567$     d)  $\sin \theta = 3.1125$

□ Remember that an inverse function □ undoes □ what a function does.

□ Our calculator uses standard inverse function notation for this.

□ Look for the inverse functions above the tri□gonometric keys.

**WARNING:** Remember that “ $\text{SIN}^{-1}$ ” on your calculator stands for the **in□verse □unction** of **□sine**. Do not use it for the reciprocal of the sine function.

a) □ or **decimal degrees** □

press  $\boxed{2^{\text{nd}}}$   $\boxed{\text{SIN}^{-1}}$   $\boxed{.}$   $\boxed{5}$   $\boxed{4}$   $\boxed{3}$   $\boxed{2}$   $\boxed{)}$   $\boxed{\text{ENTER}}$ .

□ or **degrees, minutes, seconds**,

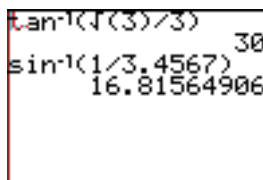
press  $\boxed{2^{\text{nd}}}$   $\boxed{\text{SIN}^{-1}}$   $\boxed{.}$   $\boxed{5}$   $\boxed{4}$   $\boxed{3}$   $\boxed{2}$   $\boxed{)}$

$\boxed{2^{\text{nd}}}$   $\boxed{\text{ANGLE}}$   $\boxed{4}$  (for convert to de□rees □minutes □seconds)  $\boxed{\text{ENTER}}$ .

$$\theta \approx 32.9^\circ \approx 32^\circ 54' 6''$$



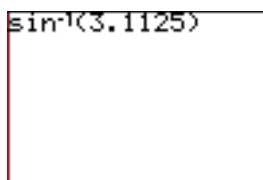
- b) Be sure to correctly group the radical and the fraction.  
As we know from our exact values  $\theta = 30^\circ$



tan<sup>-1</sup>(√(3)/3) 30  
sin<sup>-1</sup>(1/3.4567) 16.81564906

- c) Remember that the cosecant and sine functions are reciprocals.

So if  $\csc \theta = 3.4567$  then  $\sin \theta = \frac{1}{3.4567}$ .  
 $\theta \approx 16.8^\circ$



sin<sup>-1</sup>(3.1125)

- d) Remember that sine values are in the range  $[-1, 1]$ .  
So this is not possible.  
If you try it on your calculator you will get a domain error message.



ERR:DOMAIN  
1:Quit  
2:Goto

Press **ENTER** to return to the home screen.

There is no angle for which  $\sin \theta = 3.1125$ .

### Finding Other Angles in Degrees

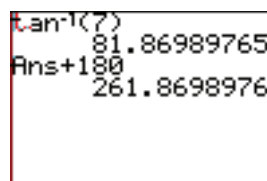
We can also find angles in other quadrants.  
First find the reference angle using your calculator.  
Then adjust for the quadrant.

**Example** Approximate  $\theta$ , where  $0^\circ \leq \theta < 360^\circ$ , given the following:

- a)  $\tan \theta = 7$  and  $\theta$  terminates in quadrant III  
Find the reference angle  $\bar{\theta}$ ,  
using your calculator.

$\bar{\theta} \approx 81.9^\circ$

Press **+** **1** **8** **0** **ENTER** to find the angle in quadrant III.



tan<sup>-1</sup>(7) 81.86989765  
Ans+180 261.8698976

$\theta \approx 261.9^\circ$

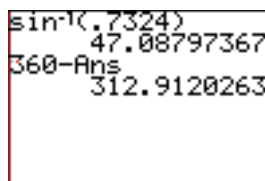
b)  $\sin \theta = -0.7324$  and  $\theta$  terminates in quadrant IV

Use a **positive** value to find  $\bar{\theta}$ .

$$\bar{\theta} \approx 47.1^\circ$$

Press  $\boxed{3} \boxed{6} \boxed{0} \boxed{-} \boxed{2^{\text{nd}}} \boxed{\text{ANS}}$

(above the negative key)  $\boxed{\text{ENTER}}$ .



$$\theta \approx 312.9^\circ$$

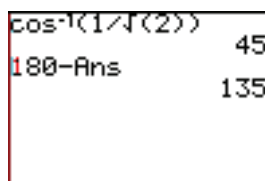
WARNING: Always enter a **positive** trigonometric value to find  $\bar{\theta}$ . If you enter a negative value you will **not** get a positive acute angle.

c)  $\sec \theta = -\sqrt{2}$  and  $\theta$  terminates in quadrant II

Remember that  $\cos \theta = \frac{1}{\sec \theta}$ .

$$\bar{\theta} = 45^\circ$$

Adjust for quadrant  $\boxed{11}$



$\theta = 135^\circ$  You may have known this exact value without needing your calculator.

### Finding Trigonometric Values Using Radian Mode

First set your calculator to radian mode.  
(See the first topic in this section.)



**Example** Use your calculator to approximate the following.

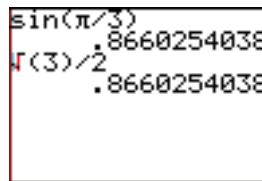
a)  $\sin\left(\frac{\pi}{3}\right)$  b)  $\cos\left(\frac{7\pi}{5}\right)$  c)  $\tan(-30)$  d)  $\tan\left(\frac{\pi}{2}\right)$

a) Press  $\boxed{\text{SIN}} \boxed{2^{\text{nd}}} \boxed{\pi} \boxed{\div} \boxed{3} \boxed{)} \boxed{\text{ENTER}}$ .

$$\sin\left(\frac{\pi}{3}\right) \approx 0.8660$$

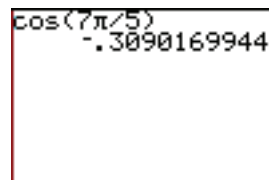
Did you recognize this as an

approximation of  $\frac{\sqrt{3}}{2}$ ?



b) Press  $\boxed{\text{COS}} \boxed{7} \boxed{2^{\text{nd}}} \boxed{\pi} \boxed{\div} \boxed{5} \boxed{)} \boxed{\text{ENTER}}$ .

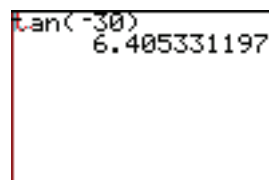
$$\cos\left(\frac{7\pi}{5}\right) \approx -0.3090$$



c) Press  $\boxed{\text{TAN}} \boxed{(-)} \boxed{3} \boxed{0} \boxed{)} \boxed{\text{ENTER}}$  (negative sign to the ENTER key).

Notice that your calculator can evaluate trigonometric functions at negative angles.  
Note also that this is radians not degrees.

$$\tan(-30) \approx 6.4053$$



WARNING: Be sure to use the **negative key**.  
Do not use the subtract key for a negative.

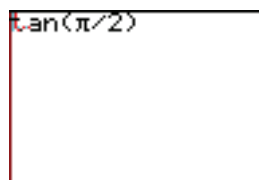
WARNING: Notice that this is the tangent of  $-30$  **radians**, not degrees.

d) Press  $\boxed{\text{TAN}} \boxed{2^{\text{nd}}} \boxed{\pi} \boxed{\div} \boxed{2} \boxed{)} \boxed{\text{ENTER}}$ .

Your calculator gives you an error message, because the value of this quadrantal angle is undefined.

Press  $\boxed{\text{ENTER}}$  to return to the home screen.

$$\tan\left(\frac{\pi}{2}\right) \text{ is undefined.}$$




### Finding Values for Secant, Cosecant and Cotangent using Radian Mode

You may have noticed that your calculator does not have keys for secant, cosecant and cotangent.

Use the reciprocal identities to find values for secant, cosecant and cotangent.

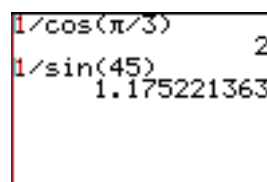
Use your calculator to approximate the following.

a)  $\sec\left(\frac{\pi}{3}\right)$    b)  $\csc(45)$    c)  $\cot\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$    d)  $\cot\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{4}\right)$

a)  $\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)}$

Press  $\boxed{1} \boxed{\div} \boxed{\cos} \boxed{2} \boxed{nd} \boxed{\pi} \boxed{\div} \boxed{3} \boxed{)} \boxed{\text{ENTER}}$

$\sec\left(\frac{\pi}{3}\right) = 2$



1/cos(pi/3) 2  
1/sin(45) 1.175221363

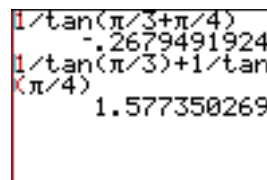
b)  $\csc(45) = \frac{1}{\sin(45)}$

$\csc(45) \approx 1.1752$

*Be careful with the order of operations.*

c)  $\cot\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)}$

$\cot\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \approx -0.2679$



1/tan(pi/3+pi/4) -.2679491924  
1/tan(pi/3)+1/tan(pi/4) 1.577350269

d)  $\cot\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} + \frac{1}{\tan\left(\frac{\pi}{4}\right)}$

$\cot\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{4}\right) \approx 1.5774$

### Finding Acute Angles in Radians

You can use your calculator to approximate an acute angle  $\theta$ , given a trigonometric value.

Remember that an acute angle in radians is an angle between 0 and  $\frac{\pi}{2} \approx 1.5708$ .

**Example** Find an approximate acute angle  $\theta$  in radians, given the following:

a)  $\sin \theta = 0.5$     b)  $\tan \theta = \frac{1}{2}$     c)  $\csc \theta = 3.4567$     d)  $\cos \theta = 2.1798$

Remember that an inverse function *undoes* what a function does.

Your calculator uses standard inverse function notation for this.

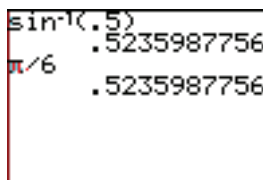
Look for the inverse functions above the trigonometric keys.

**WARNING:** Remember that “ $\text{SIN}^{-1}$ ” on your calculator stands for the **inverse function** of **sine**. Do not use it for the reciprocal of the sine function.

a) Press  $2^{\text{nd}}$   $\text{SIN}^{-1}$   $.$   $5$   $)$   $\text{ENTER}$ , for radians.

$$\theta \approx 0.5236$$

Did you recognise this as  $\frac{\pi}{6}$ ?



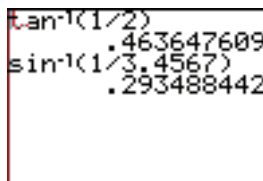
b) Be sure to correctly group the fraction.

$$\theta \approx 0.4636$$

c) Remember that the cosecant and sine functions are reciprocals.

So if  $\csc \theta = 3.4567$ , then  $\sin \theta = \frac{1}{3.4567}$ .

$$\theta \approx 0.2935$$

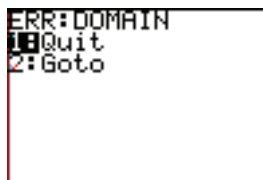
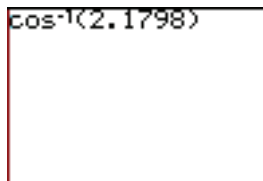


d) Remember that sine values are in the range  $[-1, 1]$ .

So this is not possible.

If you try it on your calculator, you will get a domain error message.

There is no angle for which  $\cos \theta = 2.1798$ .



Finding Other Angles in Radians

We can also find angles in other quadrants.

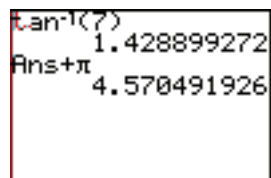
First find the reference angle using your calculator. Then adjust for the quadrant.

**Example** Approximate  $\theta$ , where  $0 \leq \theta < 2\pi$ , given the following:

a)  $\tan \theta = 7$  and  $\theta$  terminates in quadrant III

Find the reference angle,  $\bar{\theta}$ , using your calculator.

$$\bar{\theta} \approx 1.4289$$



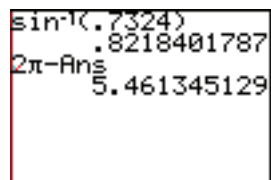
Press  $+$   $\pi$   $\text{ENTER}$  to find the angle in quadrant III

$$\theta \approx 4.5705$$

b)  $\sin \theta = -0.7324$  and  $\theta$  terminates in quadrant IV

Use a **positive** value to find  $\bar{\theta}$ .

$$\bar{\theta} \approx 0.8218$$



Press  $2$   $\pi$   $-$   $2^{\text{nd}}$   $\text{ANS}$

above the negative key  $\text{ENTER}$ .

$$\theta \approx 5.4613$$

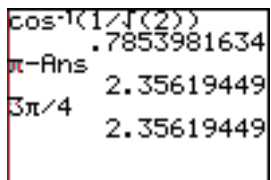
**WARNING:** Always enter a **positive** trigonometric value to find  $\bar{\theta}$ . If you enter a negative value you will not get a positive acute angle.

c)  $\sec \theta = -\sqrt{2}$  and  $\theta$  terminates in quadrant II

Remember that  $\cos \theta = \frac{1}{\sec \theta}$ .  $\bar{\theta} \approx 0.7854$

Adjust for quadrant II

$$\theta \approx 2.3562$$







Did you recognize this as an approximation for  $\frac{3\pi}{4}$ ?

### Evaluating a Trigonometric Function in Degrees While in Radian Mode

You can use degrees while in radian mode by using the degree symbol.

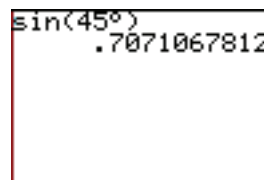
  Find  $\sin 45^\circ$  while the calculator is in radian mode.



Press     ANGLE



 above the  

 for degree  .



### Evaluating a Trigonometric Function in Radians While in Degree Mode

You can use radians while in degree mode by using the radian symbol.

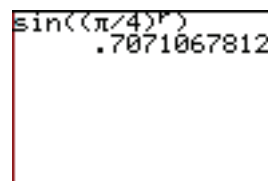
  Find  $\sin\left(\frac{\pi}{4}\right)$  while the calculator is in radian mode.



Press       

 ANGLE  above the  

 for radian  .



WARNING: Be sure to group the radians in parenthesis before using the radian symbol so that all of the value is designated as radians.

### Graphing the Trigonometric (Circular) Functions in Radian Mode



First, be sure that your calculator is in radian mode.

Then, as with any graph, consider the scale you want to use and an appropriate window.

The ZTrig zoom option is designed to graph the six standard trigonometric functions.



ZTrig in Radian Mode

The seventh zoom option is the zoom trigonometry window. Select it and then press **WINDOW** to see the window values **in radian mode**.

```

WINDOW
Xmin=-6.152285...
Xmax=6.1522856...
Xscl=1.5707963...
Ymin=-4
Ymax=4
Yscl=1
Xres=1

```

Notice that the y-values are from  $-4$  to  $4$ .

The x-values are from about  $-6.15$  to  $6.15$ .

Notice that  $6.15$  is slightly less than  $2\pi$ .

```

ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig

```

```

2π  6.283185307

```

**WARNING:** Always carefully consider the equation you are graphing **before** you choose your viewing window.

Graphing Sinusoidal (Trigonometric) Functions

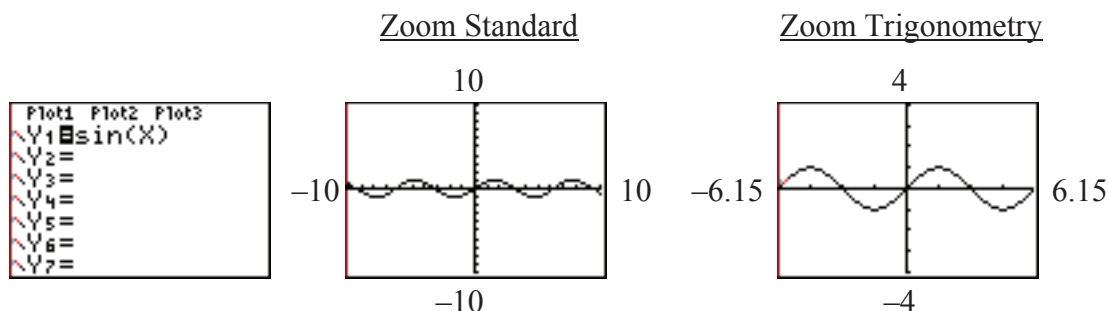
To graph a function on your calculator, press **y=**, type the function, and then graph it using an appropriate viewing window.

**Example 14** Graph  $y = \sin x$  using your graphing calculator.

Press **y=** **SIN** **x** **)**.

To graph it in a standard window press **ZOOM** **6** (for zoom standard).

The zoom trigonometry window is a better window. To use it press **ZOOM** **7**.



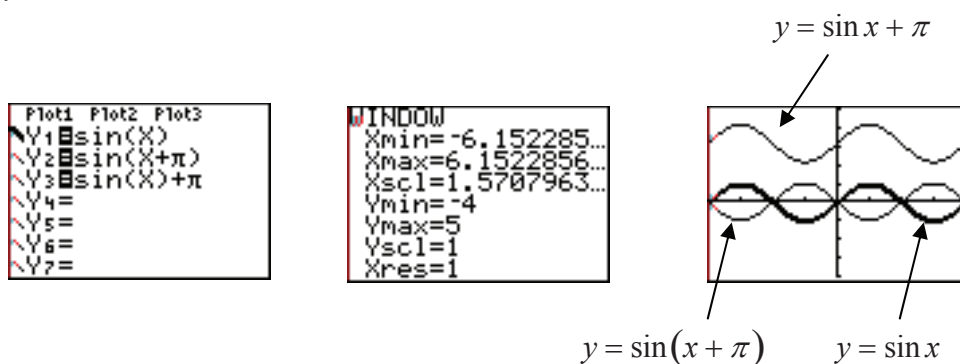
**WARNING:** Be sure that your calculator is in **radian mode**.

**Example 15** Graph  $y = \sin x$ ,  $y = \sin(x + \pi)$  and  $y = \sin x + \pi$  using your calculator.

Type each of these in the  $y=$  window. Be very careful with the parentheses.

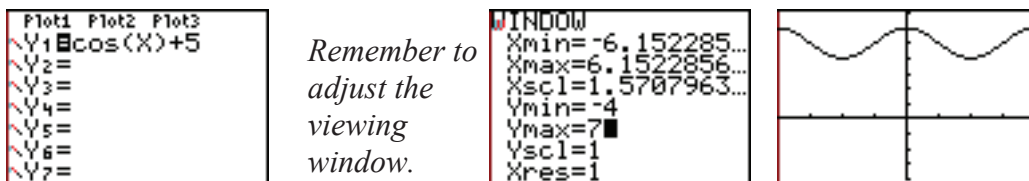
To make  $y = \sin x$  stand out, move to the left of the  $y$  in its equation and press  $\boxed{\text{ENTER}}$  once. You will now see a thick line next to the  $y$ .

Press  $\boxed{\text{WINDOW}}$  to increase the maximum  $y$ -value so that the top of the graph of  $y = \sin x + \pi$  shows on the screen.



To clear a function from the  $y=$  window, move the cursor to it and press  $\boxed{\text{CLEAR}}$ .

**Example 16** Graph  $y = \cos x + 5$  using your graphing calculator.



Remember to  
adjust the  
viewing  
window.

**Example 17** Graph  $y = -\frac{1}{2}\cos\left(\pi x + \frac{3\pi}{2}\right)$  using your graphing calculator.

**WARNING:** Be sure to use the negative key, next to the  $\boxed{\text{ENTER}}$  key, for a negative. Do not confuse it with the subtract key (in the right hand column.)

We know, from studying trigonometry, that this graph has a period of 2, a phase shift of  $-\frac{3}{2}$ , and an amplitude of  $\frac{1}{2}$ . So let's use a window where y-values go from  $-1$  to  $1$  and x-values go from  $-\frac{3}{2}$  to  $\frac{5}{2}$ .

```

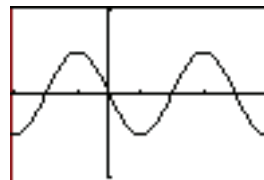
Plot1 Plot2 Plot3
Y1 (-1/2)cos(πX
+3π/2)
Y2=
Y3=
Y4=
Y5=
Y6=

```

```

WINDOW
Xmin=-1.5
Xmax=5/2
Xscl=.5
Ymin=-1
Ymax=1
Yscl=1
Xres=1

```



**WARNING:** Always be careful when you use parentheses on your graphing calculator. Compare how we used parentheses in Examples 14 – 17.

### Asymptotes and Your Calculator

**Example 18** Graph  $y = \tan x$  using your graphing calculator.

The graphs below were done using a TI-84.

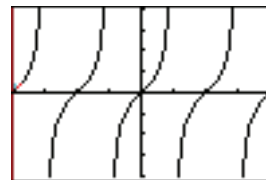
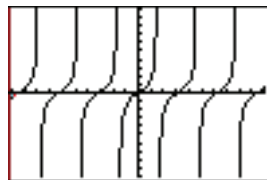
Zoom Standard

Zoom Trigonometry

```

Plot1 Plot2 Plot3
Y1 tan(X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```



The following graphs were drawn using an older TI-83 calculator. This older operating system plots points and connects them. This can make it look like the calculator is showing asymptotes, but it is not. Always consider what you know about a graph when you are interpreting these lines.

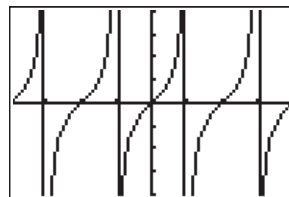
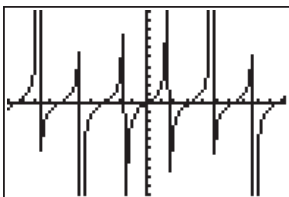
zoom standard

zoom trigonometry

```

Plot1 Plot2 Plot3
Y1 tan(X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```



**WARNING:** Your calculator **does not draw asymptotes**. Depending on the operating system, it may or may not connect points across undefined values. Use what you know about the function to help you interpret the calculator's graph.

### The Reciprocal Functions

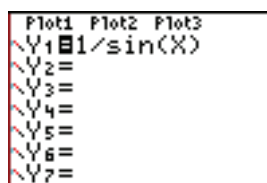
Your calculator does not have keys for the cotangent, secant and cosecant functions. Use what you know about their reciprocal relationship with the other three trigonometric functions.

**Example 19** Graph  $y = \csc \pi x$  using your graphing calculator.

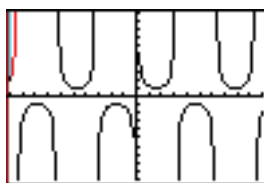
Remember that  $\csc x = \frac{1}{\sin x}$ .

*If we use a zoom standard window with a TI-84/83 Plus, there are so many connections that the graph is unclear. A better window shows  $x$ -values from 0 to 2 and  $y$ -values from  $-10$  to  $10$ .*

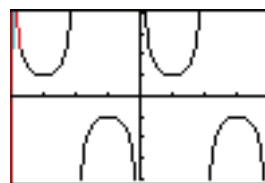
*The period for this graph is 2. Press **TRACE** **1** **ENTER** to see that the calculator knows that  $\csc \pi$  is undefined. The calculator does not connect the two parts of the curve.*



Zoom Standard



Zoom Trigonometry



**WARNING:** A given window may be better or worse depending upon the type of graphing calculator you use.

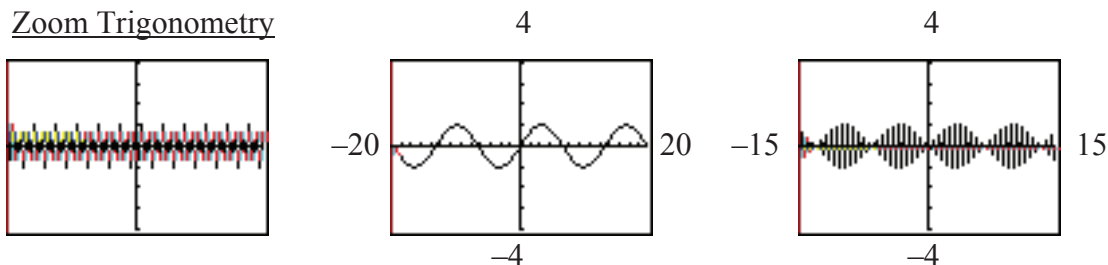
### Choosing an Appropriate Window

**WARNING:** Do not count on your calculator to automatically find a good viewing window for graphing a trigonometric equation. Find its period, phase shift, and amplitude (if possible) **before** you graph it using a calculator.

**Example 20** Graph  $y = \sin 30x$  using your graphing calculator.

*Your calculator can give you very different shapes for this graph, depending upon the window you use. Here are some incorrect graphs for this function.*

Zoom Trigonometry



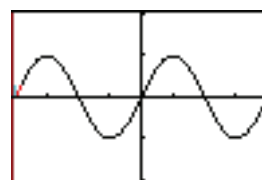
To graph this correctly, notice that the period is  $\frac{\pi}{15}$ . A good window

shows  $x$ -values from  $-\frac{\pi}{15}$  to  $\frac{\pi}{15}$ , with  $y$ -values from  $-2$  to  $2$ .

```

WINDOW
Xmin=-.2094395...
Xmax=.20943951...
Xscl=π/60
Ymin=-2
Ymax=2
Yscl=1
Xres=1

```



**WARNING:** Do not count on your calculator to automatically find a good viewing window for graphing a trigonometric function. Always find the function's period, phase shift and amplitude **before** you graph it using a calculator.

### Graphing the Trigonometric (Circular) Functions in Degree Mode

Although we rarely graph trigonometric functions in degree mode, your calculator can do so.

First be sure that your calculator is in degree mode.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SETCLOCK 07/31/09 1:55PM

```

Then, as with any graph, consider the scale you want to use and an appropriate window. The ZTrig zoom option is designed to graph the six standard trigonometric functions.

ZTrig in Degree Mode

The seventh zoom option is the zoom trigonometry window.

Select it and then press **WINDOW** to see the window values **in degree mode**.

```

WINDOW
Xmin=-352.5
Xmax=352.5
Xscl=90
Ymin=-4
Ymax=4
Yscl=1
Xres=1

```

Notice that the y-values are from  $-4$  to  $4$ .

The x-values are from about  $-352.5^\circ$  to  $352.5^\circ$ .

Notice that this range is **almost**  $-360^\circ$  to  $360^\circ$ .

```

ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig

```

**WARNING:** Always carefully consider the equation you are graphing **before** you choose your viewing window.

**Example 21** Graph  $y = \sin x$  using your calculator, in degree mode.

Press **y=** **SIN** **x** **)**.

To graph it in a standard window

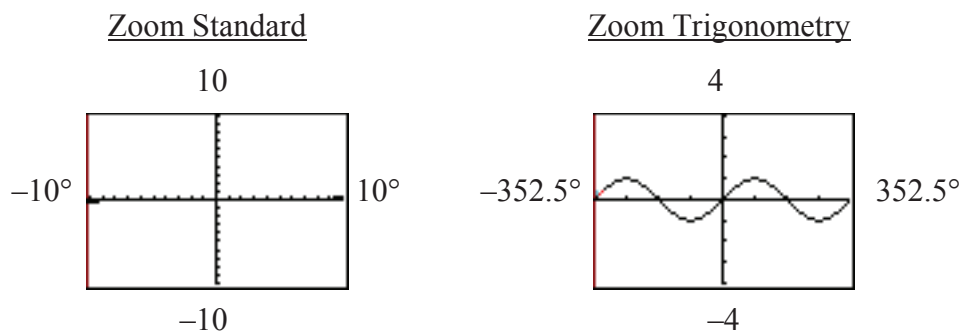
press **ZOOM** **6** (for zoom standard).

```

Plot1 Plot2 Plot3
Y1=sin(X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```

The zoom trigonometry window is a better window. To use it press **ZOOM** **7**.

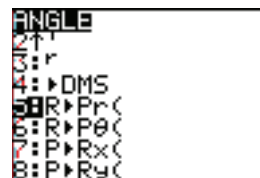


**WARNING:** Notice that a zoom standard window shows only from  $-10$  **degrees** to  $10$  **degrees** when your calculator is in **degree mode**.

## 4 Polar Coordinates and Graphs

### Using a Calculator to Convert Points

You can find the calculator commands to convert between polar and rectangular coordinates by pressing  $2^{\text{nd}}$  ANGLE . Use options 5 through 8.



R stands for rectangular and P stands for polar.

For example, option 6 stands for “find the polar  $\theta$  given the rectangular coordinates.” The result is only the  $\theta$ -coordinate.

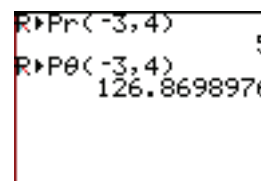
**WARNING:** Because  $\theta$  can be given in either degrees or radians, be sure that you have your calculator in the mode that you plan to use.

**Example 1** Find the polar coordinates in degrees for the rectangular point  $(-3, 4)$ .

Be sure that your calculator is in **degree mode**.

Press  $2^{\text{nd}}$  ANGLE  $5$  for option 5, rectangular coordinates to polar  $r$ , then  $(-)$   $3$  , (comma, the key above the 7 key)  $4$  )  $\text{ENTER}$  to find  $r$ .

Press  $2^{\text{nd}}$  ANGLE  $6$  for option 6, rectangular coordinates to polar  $\theta$ , then  $(-)$   $3$  ,  $4$  )  $\text{ENTER}$  to find  $\theta$ .



The polar coordinates for the rectangular point  $(-3, 4)$  are approximately  $(5, 126.9^\circ)$ .

**Example 2** Find the polar coordinates in radians for the rectangular point  $(2, -2)$ .

Be sure that your calculator is in **radian mode**.

Press  $2^{nd}$  ANGLE  $5$

for option 5, rectangular coordinates to polar  $r$ ,

then  $2$  ,  $(-)$   $2$  )  $\text{ENTER}$  to find  $r$ .

Press  $2^{nd}$  ANGLE  $6$

for option 6, rectangular coordinates to polar  $\theta$ ,

then  $2$  ,  $(-)$   $2$  )  $\text{ENTER}$  to find  $\theta$ .

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SETCLOCK08/05/09 11:14AM
```

```
R→Pr(2, -2)
2.828427125
R→Pθ(2, -2)
-.7853981634
```

Notice that  $\theta$  is an approximation for  $-\frac{\pi}{4}$ .

The polar coordinates for the rectangular point  $(2, -2)$  are approximately  $(2.83, -0.79)$ .

WARNING: Your calculator yields only **one** set of polar coordinates. Remember that, although a point has only one set of rectangular coordinates, it has infinitely any different sets of polar coordinates. For example, the rectangular point  $(2, -2)$  can be found using the polar coordinates  $\left(2, \frac{7\pi}{4}\right)$ ,  $\left(-2, \frac{3\pi}{4}\right)$ ,  $\left(-2, -\frac{5\pi}{4}\right)$  and  $\left(2, -\frac{\pi}{4}\right)$ .

**Example 3** Find the rectangular coordinates for the polar point  $(-6, 330^\circ)$

Be sure that your calculator is in **degree mode**.

Press  $2^{nd}$  ANGLE  $7$

for option 7, polar coordinates to rectangular  $x$ ,

then  $(-)$   $6$  ,  $330$  )  $\text{ENTER}$  to find  $x$ .

Press  $2^{nd}$  ANGLE  $8$

for option 8, polar coordinates to rectangular  $y$ ,

then  $(-)$   $6$  ,  $330$  )  $\text{ENTER}$  to find  $\theta$ .

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SETCLOCK08/05/09 11:12AM
```

```
P→Rx(-6, 330)
-5.196152423
P→Ry(-6, 330)
3
```

The rectangular coordinates for the polar point  $(-6, 330^\circ)$  are approximately  $(-5.20, 3)$ .



**Example 4** Find the rectangular coordinates for the polar point  $\left(2, \frac{\pi}{4}\right)$ .

Be sure that your calculator is in **radian mode**.

Press  $2^{nd}$  ANGLE  $7$

for option 7, polar coordinates to rectangular  $x$ ,  
then  $2$  ,  $2^{nd}$   $\pi$   $\div$   $4$   $)$   $\text{ENTER}$  to find  $x$ .

Press  $2^{nd}$  ANGLE  $8$

for option 8, polar coordinates to rectangular  $y$ ,  
then  $2$  ,  $2^{nd}$   $\pi$   $\div$   $4$   $)$   $\text{ENTER}$  to find  $y$ .

The rectangular coordinates for the polar point  $\left(2, \frac{\pi}{4}\right)$  are approximately  $(1.41, 1.41)$ .

### Graphing Polar Equations

To graph a polar equation using your calculator, first change to polar mode. Press  $\text{MODE}$ , move down and across to POL and press  $\text{ENTER}$  to select it. You can use either degree or radian mode.

In polar mode, the  $x$ -key becomes the  $\theta$ -key.

**WARNING:** Remember to change back to FUNC (function mode) when you want to work with functions of  $x$ .

### The Zoom-Standard Polar Window

In polar mode,  $\text{ZOOM}$   $6$ , ZStandard, graphs  $\theta$  from 0 to approximately  $2\pi$  in radian mode and  $0^\circ$  to  $180^\circ$  in degree mode, with  $x$  and  $y$  from  $-10$  to  $10$ .

#### Radian mode

#### Degree mode

### Adjusting the Window Values

As in rectangular graphing, you can adjust the window values by pressing  $\text{WINDOW}$  and entering new values.

Polar Graph Coordinates

You calculator can tell you the rectangular coordinates of a point on a curve, or the polar coordinates.

To change from one to the other,

press  $2^{ND}$  FORMAT (above  $ZOOM$ ),

move on top of either RectGC  
(for rectangular graph coordinates)

or PolarGC

(for polar graph coordinates) and

press  $\square$  to select it.

Then return to your graph or the home screen.

```
RectGC PolarGC
CoordOn CoordOff
GridOff GridOn
AxesOn AxesOff
LabelOff LabelOn
ExprOn ExprOff
```

```
RectGC PolarGC
CoordOn CoordOff
GridOff GridOn
AxesOn AxesOff
LabelOff LabelOn
ExprOn ExprOff
```

WARNING: Be sure to change back to rectangular graph coordinates when done.

Polar Graphing Examples

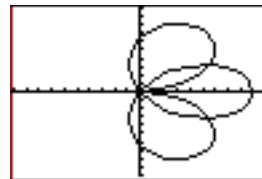
**Example 5** Graph  $r = 9 \sin\left(\frac{3}{2}\theta\right)$ .

Be sure that your calculator is in polar mode and radian mode.

Press  $\square$ . Notice that it now says  $r=$ .

Type the equation using the  $x$ -key to enter  $\theta$ .

```
Plot1 Plot2 Plot3
r1=9sin(3θ/2)
r2=
r3=
r4=
r5=
r6=
```



To graph it in a standard window

press  $ZOOM$   $\square$  (for zoom standard).

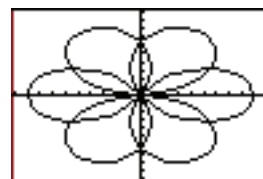
Notice that this does not look complete.

The standard window graphs  $\theta$ -values from 0 to  $2\pi$ .

To show all of this rose, we need  $\theta$ -values from 0 to  $4\pi$ .

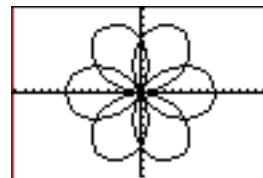
Press  $WINDOW$  and adjust the  $\theta$ -max to  $4\pi$ .

```
WINDOW
θmin=0
θmax=4π
θstep=.1308996...
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
```



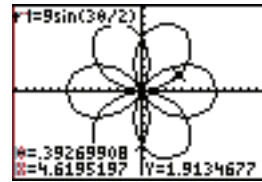
Now we can see all of the petals of the rose, but it looks misshapen because the screen is wider than it is tall. To adjust for this,

press  $ZOOM$   $\square$  (for zoom square).



Press **TRACE**.

Move the right or left cursor  
to see **rectangular** coordinates on the curve.

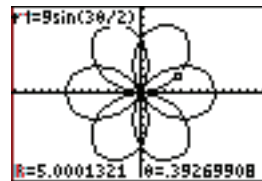


Press **2<sup>nd</sup>** **FORMAT** to change  
to polar graph coordinates.



Move to the right and press **ENTER** to select it.  
Then press **TRACE**.

Move the right or left cursor  
to see **polar** coordinates on the curve.

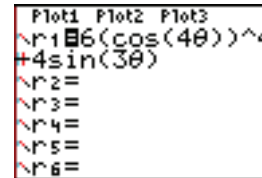


**Example 7** Use your graphing calculator to graph  $r = 6 \cos^4 4\theta + 4 \sin 3\theta$ .

Be sure that your calculator is in polar mode and radian mode.

Press **y=**. Notice that it now says  $r=$ .

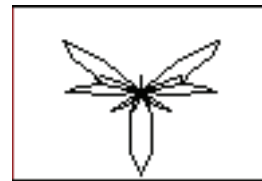
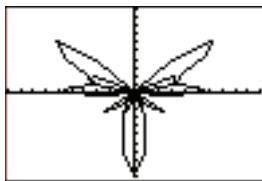
Type the equation.



Press **ZOOM** **6** to graph it in a standard window.

To eliminate the axes (and better see the curve), press **2<sup>nd</sup>** **FORMAT** (above **ZOOM**).

Select **AxesOff** and press **GRAPH** to return to the graph.



Press **2<sup>nd</sup>** **FORMAT**, select **AxesOn** and press **GRAPH** to return the axes to the graph.

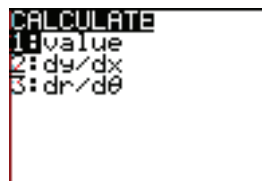
Finding a Value

You can use the **TRACE** key to find an  $r$ -value when your calculator is in polar graph coordinates. Press **TRACE**, type in the  $\theta$ -value and press **ENTER**.

You can also find  $r$ -values by pressing

**2<sup>nd</sup>** **CALCULATE** (above **TRACE**)

**1** type in the  $\theta$ -value and press **ENTER**.



**Example 7** Assume  $r = 6 \cos^4 4\theta + 4 \sin 3\theta$ . Find  $r$  if  $\theta = 70^\circ$ .

Be sure that your calculator is in polar mode.

If you graphed this in Example 6 above,

the equation is already in your calculator.

If not, follow the directions in Example 6.

You can have the axes on or off as you wish.

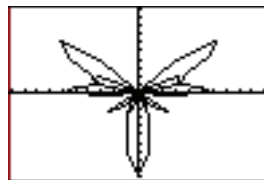
Be sure that the coordinates are set to **polar graph coordinates**.



Because  $\theta$  is in **degrees**, change to degree mode.



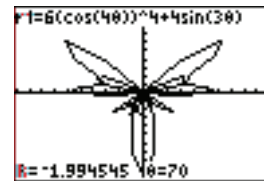
Then press **ZOOM** **6** to graph it in a standard **degree** window.



Then press **TRACE** **7** **0** **ENTER**

OR, press **2<sup>nd</sup>** **CALCULATE** (above **TRACE**)

**1** **7** **0** **ENTER**.



If  $\theta = 70^\circ$ , then  $r$  is approximately  $-2$ .

Using a Polar Table

We can use a table to see many polar function values at a time.

Enter the function in the  $y=$  screen.

Press  $2^{\text{nd}}$  TBLSET to set a starting  $\theta$ -value and  $\Delta\text{Tbl}$ , the difference between  $\theta$ -values.

Set both Indpnt and Depend (independent and dependent variable) to Auto.

Press  $2^{\text{nd}}$  TABLE to see the values in the table.

**Example 8** Use a table to show values for  $r = 6 \cos^4 4\theta + 4 \sin 3\theta$  in degrees.

*Be sure that your calculator is in polar mode and degree mode.*

*Type the function into the  $y=$  screen.*

*Before viewing the table, we need to use the Table Setup Window to enter a starting  $\theta$ -value and  $\Delta\text{Tbl}$ , the difference between each  $\theta$ -value.*

*Let's start at  $0^\circ$  and increase in steps of  $15^\circ$ .*

Press  $2^{\text{nd}}$  TBLSET (for table setup, above WINDOW).

Press  $0$  ENTER  $1$   $5$ .

TABLE SETUP  
TblStart=0  
 $\Delta\text{Tbl}=15$   
Indpnt: Auto Ask  
Depend: Auto Ask

$\theta$	r1	
0	6	
15	3.2034	
30	4.375	
45	0.8284	
60	-.375	
75	-2.453	
90	2	

*To see the table:*

Press  $2^{\text{nd}}$  TABLE (above GRAPH)

You can use the up and down cursor arrows to scroll through any table.

You can use the Ask feature to find values in our table.

In the table setup screen, change Auto to Ask.

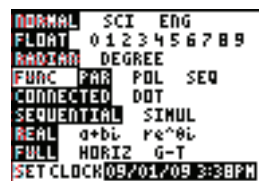
## 5 Parametric Equations

### Graphing Parametric Equations

To graph parametric equations using your calculator, first change to parametric mode. Press **MODE**, move down and across to PAR and press **ENTER** to select it.

You can use either degree or radian mode.

In parametric mode, the  $x$ -key becomes the  $t$ -key.

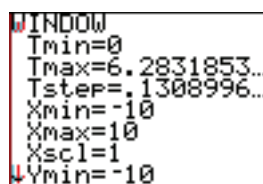


**WARNING:** Remember to change back to FUNC (function mode) when you want to work with functions of  $x$ .

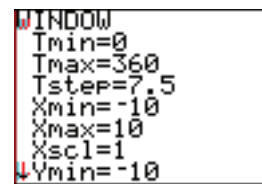
### The Zoom-Standard Parametric Window

In parametric mode, **ZOOM** **6**, ZStandard, graphs  $t$  from 0 to approximately  $2\pi$  in radian mode and  $0^\circ$  to  $360^\circ$  in degree mode, with  $x$  and  $y$  from  $-10$  to  $10$ .

#### Radian mode



#### Degree mode



### Adjusting the Window Values

As in rectangular graphing, you can adjust the window values by pressing **WINDOW** and entering new values.

### Parametric Graphing Examples

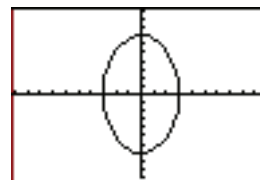
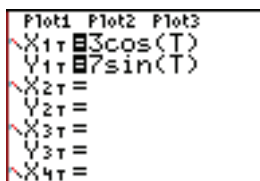
**Example 1** Graph  $\begin{cases} x = 3 \cos t \\ y = 7 \sin t \end{cases}$  using your graphing calculator.

Change to parametric mode. You can graph this in either degree or radian mode. Let's use radian mode.

Press **y=**. Notice that the window is set up for parametric equations.

Type the equations.

Press **ZOOM** **6** (zoom standard) to see the ellipse in a standard window.



**Example** □ Graph  $\begin{cases} x = 4.2 \cos(10\pi t) \\ y = 4.2 \sin(10\pi t) \end{cases}$  using your graphing calculator.

Be sure that you are in **parametric** and **radian** mode.

Press  $\boxed{Y=}$  and type the equations.

If you graph this in a standard window it will not give a correct graph. This is because the period of one fifth is almost as small as the standard  $t$ -step.

Change to the  $t$ -values shown below and press  $\boxed{\text{ENTER}}$ .

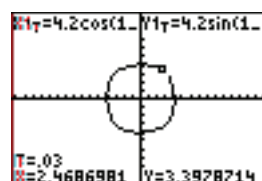
Then press  $\boxed{\text{ZOOM}}$   $\boxed{\square}$  (square) for a window that uses the same scale on each axis. This will make the circle look round.

Watch the screen as it graphs to see the direction of the curve.

Or press  $\boxed{\text{TRACE}}$ . Then use the cursor to follow the path as  $t$  increases

```
Plot1 Plot2 Plot3
X1t=4.2cos(10πT
Y1t=4.2sin(10πT
X2t=
Y2t=
X3t=
```

```
WINDOW
Tmin=0
Tmax=1
Tstep=.01
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
```



**Example** □ A child throws a ball from a height four feet above the ground. She throws it at an angle of  $45^\circ$  from the ground with a velocity of  $16\sqrt{2}$  ft/s (about 22.63 ft/s). Assume that the child throws the ball at time  $t = 0$ .

The parametric equations that model this are  $\begin{cases} x = 16t \\ y = 16t - 16t^2 + 4 \end{cases}$ .

Graph the trajectory (the path) of the ball on your graphing calculator.

Looking at a standard window in radian mode we can see that we need to increase the  $x$ -max value.

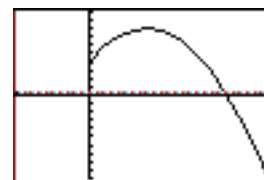
Use window to change  $x$ -max to 25 so that we can see where the ball hits the ground.

Remember that the ball stops when it hits the ground (the  $x$ -axis).

(If you want to use degree mode set  $T$ -step to  $\pi$  with a step of  $\pi$ )

Finding a Value

```
WINDOW
Tmin=0
Tmax=6.2831853...
Tstep=.1308996...
Xmin=-10
Xmax=25
Xscl=1
Ymin=-10
```



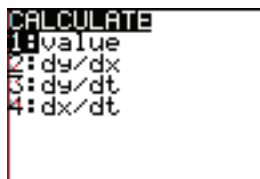
You can use the  $\boxed{\text{F2}}$   $\boxed{\text{F1}}$   $\boxed{\text{F2}}$  to find  $x$ - and  $y$ -values for a given  $t$ -value, if your calculator is in parametric mode.

Press  $\boxed{\text{TRACE}}$ , type in the  $t$ -value and press  $\boxed{\text{ENTER}}$ .

You can also find  $r$ -values by pressing

$\boxed{\text{F2}}$   $\boxed{\text{CALCULATE}}$  (above  $\boxed{\text{TRACE}}$ )

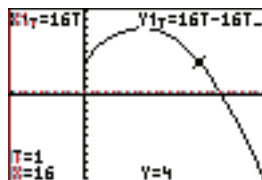
$\boxed{\text{F1}}$  and type in the  $t$ -value and press  $\boxed{\text{ENTER}}$ .



**Example 1** Find the location of the ball from Example 3 after one second.

Graph the parametric equations as in Example 1.

Then press  $\boxed{\text{TRACE}}$   $\boxed{\text{F1}}$   $\boxed{\text{ENTER}}$ .



### Using a Parametric Table

We can use a table to see many parametric equation values at a time.

Enter the function in the  $y=$  screen.

Press  $\boxed{\text{F2}}$   $\boxed{\text{TBASE}}$  to set a starting  $t$ -value and  $\Delta T$ , the difference between  $t$ -values.

Set both  $\boxed{\text{I/O}}$  and  $\boxed{\text{D/O}}$  (independent and dependent variable) to  $\boxed{\text{A}}$ .

Press  $\boxed{\text{F2}}$   $\boxed{\text{TABLE}}$  to see the values in the table.

**Example 2** Use a table to show values for the trajectory of the ball in Example 3.

Be sure that your calculator is in parametric mode and radian mode.

Type the function into the  $y=$  screen.

Before viewing the table, we need to use the Table Setup window to enter a starting  $t$ -value and  $\Delta T$ , the difference between each  $t$ -value.

Let's start at 0 and increase in steps of  $\frac{1}{4}$ .



Press  $\boxed{\text{2ND}} \boxed{\text{T}} \boxed{\text{MODE}} \boxed{\text{ET}}$  (for table setup)  $\boxed{\text{a}} \boxed{\text{b}} \boxed{\text{c}} \boxed{\text{WINDOW}}$ .

Press  $\boxed{\text{2ND}} \boxed{\text{ENTER}} \boxed{\text{2ND}} \boxed{\text{2ND}} \boxed{\text{2ND}}$ .

```
TABLE SETUP
TblStart=0
ΔTbl=1/4
Indent: Auto Ask
Depend: Auto Ask
```

T	X <sub>1T</sub>	Y <sub>1T</sub>
0	0	4
.25	4	3.75
.5	8	3
.75	12	1.75
1	16	-1
1.25	20	-4
1.5	24	-9

To see the table

Press  $\boxed{\text{2ND}} \boxed{\text{TA}} \boxed{\text{MODE}} \boxed{\text{a}} \boxed{\text{b}} \boxed{\text{c}} \boxed{\text{RAP}}$

Notice that the ball hits the ground between 1 and 1.5 seconds after it is thrown.

You can use the up and down cursor arrows to scroll through any table.

You can use the  $\boxed{\text{A}}$  feature to find values in our table.

In the table setup screen, change  $\boxed{\text{A}}$  to  $\boxed{\text{A}}$ .

## 6 Solving Equations Using a Graphing Calculator

---

There are several methods you can use to solve an equation using your calculator. Here are some of them.

### Solving One-Variable Equations by Graphing

Once you know how to graph using a graphing calculator (See Section 2.), you can use the calculator to solve equations and inequalities that have one variable. These calculator techniques are especially useful when the numbers in an equation are complicated, or when our other techniques for solving equations have failed.

We can solve equations that have one variable using the graphing capabilities of the TI-84/83 Plus, using the INTERSECT or ZERO commands. But first we need to understand how to use a graph in two-space to solve an equation that has only one variable.

Whether or not you're using a graphing calculator, you can solve a one-variable equation using graphing in two-space. Two-space has two variables. So we need to change the single equation into a system of equations that has two variables and two equations.

### Procedure: Solving an Equation in One Variable by Graphing

1. Rewrite the equation in the form  $f(x) = 0$ .
2. Let  $y_1 = f(x)$  and  $y_2 = 0$ . (Notice that  $y_2 = 0$  is the  $x$ -axis.)
3. Graph  $y_1$  and  $y_2$  in an appropriate window.
4. The  $x$ -coordinate of each ordered pair where  $y_1$  intersects the  $x$ -axis solves the equation  $f(x) = 0$ .

Why does this work?

If you substitute zero for  $y$  into the system of equations  $\begin{cases} y = f(x) \\ y = 0 \end{cases}$  then you get  $f(x) = 0$ .

The equation  $f(x) = 0$  is our original equation.

So the  **$x$ -coordinates** of the solutions to the **system** also solve our original **equation**.

**WARNING:** This method finds only **real solutions**. It does **not** find **imaginary solutions**.

Use a graph to solve  $x^2 = 4$ .

$$f(x) = 0 \quad x^2 - 4 = 0$$

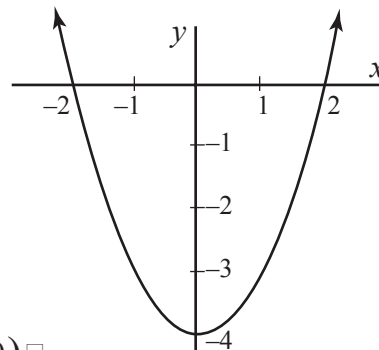
$$\begin{cases} y_1 = x^2 - 4 \\ y_2 = 0 \end{cases}$$

$y_1$

$y_2$   $x$   $x$

$(2, 0)$   $(-2, 0)$

$x$



So the solutions are  $x = 2$  or  $x = -2$ .

There are two ways that you can use your calculator to find the  $x$ -coordinates where the curve  $y = f(x)$  intersects the  $x$ -axis: (1) using INTERSECT and (2) using ZERO.

### Using INTERSECT to Solve an Equation in One Variable

The intersect command finds the intersection of two curves.

WARNING: You must have TWO curves to use INTERSECT. So remember to type in  $y_1$  and  $y_2$ .

WARNING: Your calculator uses an iterative, numeric technique. So be aware that your answers are **approximate**, not exact. And this technique does **not** give any imaginary roots.

To find the intersect command:

$\text{C} \rightarrow \text{C} \rightarrow \text{TR} \rightarrow \text{CE}$

$\text{C} \rightarrow \text{C} \rightarrow \text{C}$

```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

WARNING: We are working with functions of  $x$ . Be sure that your calculator is in **function** mode.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SETCLOCK09/01/09 5:09PM
  
```

**Example** Solve using the intersect command on your calculator.

$$x^3 + 3x^2 = x + 6$$

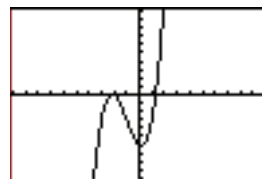
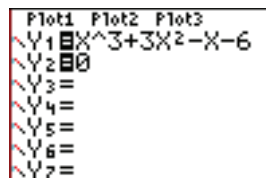
Round your answer to three decimal places.

$f(x) = 0 \quad x^3 + 3x^2 - x - 6 = 0$

$$\begin{cases} y_1 = x^3 + 3x^2 - x - 6 \\ y_2 = 0 \end{cases}$$

$y = f(x)$

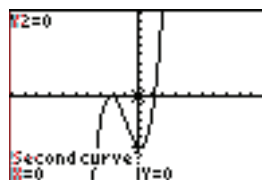
$y = 0$



$y_1 = x^3 + 3x^2 - x - 6$

$y_1 = f(x)$  **ENTER**  $y_1$

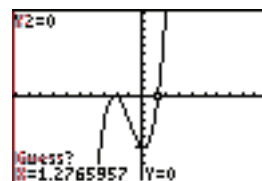
**ENTER**  $y_2$



$y = f(x)$

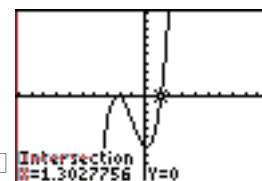
$y = f(x)$

**ENTER**



$y = f(x)$

$y = f(x)$



$x = 1.303$

**WARNING:** Be sure that it says “Intersection”

If it says “Guess” then you have not yet found the solution.

$f(x) = x^3 + 3x^2 - x - 6$





This is an example of a window that allows you to find the third intersection.



As before, our solutions are  $x = -0.861$ ,  $x = 1.240$ , or  $x = 16.000$ .

Although this approach works, it is usually easier to find the intersections of  $y_1 = x^4 - 2^x$  and  $y_2 = 0$ , because the intersections are **on the x-axis**.

When the intersections are on the x-axis, you only need to look for intersections along that horizontal line, rather than searching throughout the coordinate system.

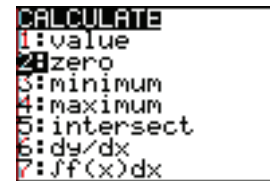
## Using ZERO to Solve an Equation in One Variable

We can use the zero command on the TI-84/83 Plus to solve equations with one variable.

This commands **assumes** that the equation is in the form  $f(x)=0$ .

It finds the **approximate real**  $x$ -values that solve the equation.

Let  $y_1 = f(x)$  and do **not** type in a  $y_2$ .



WARNING: You **must** isolate zero in your equation to use the zero command.

**am** Use the zero command on your graphing calculator to solve  $x^3 = x + 3$ . Round your answer to three decimal places.

$$x^3 - x - 3 = 0$$

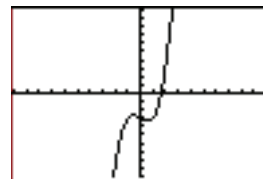
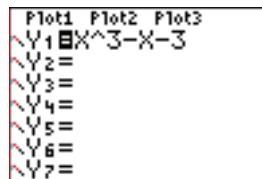
$$\boxed{\phantom{0}}\boxed{\phantom{0}}\boxed{\phantom{0}} y_1 = x^3 - x - 3.$$

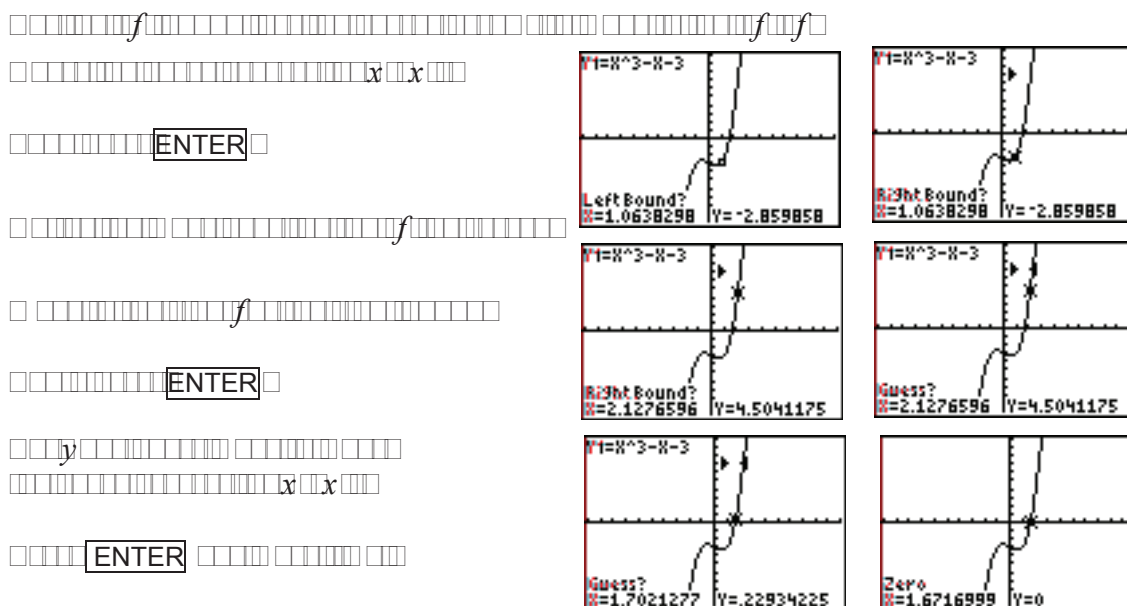
[illegible]

A horizontal number line with arrows at both ends. It is marked with numbers from 0 to 100 in increments of 10: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. A red dot is placed on the number 40.

[illegible]

A horizontal bar representing a 16-bit register. The word "ZERO" is centered within the bar. The bar is divided into segments by vertical lines, with a larger central segment containing the text "ZERO".





The real solution is approximately  $x = 1.672$ .

Calculator screen capture showing the final solution:  $x = 1.6716999$ .

**WARNING:** Be sure that it says “ $x =$ ” in the window.  
If it says “ $x \approx$ ” then you have not yet found the solution.

**WARNING:** This method finds approximate **real solutions**.  
It does **not** find **imaginary solutions**.

### Using Your Graphing Calculator to Solve an Inequality

You can use the graphing feature of your calculator to solve inequalities, using the following step:

1. Let  $y_1$  be the left side of the inequality and  $y_2$  be the right side.
2. Graph  $y_1$  and  $y_2$  in an appropriate window so that all intersections can be seen.
3. Use INTERSECT to find all of the places where the curves cross each other. These are the endpoint values.
4. Look at the graphs of the two functions. The function values are larger as you go up and smaller as you go down. So the larger function is above the smaller function. Shade the regions that match the direction of the inequality remembering that  $y_1$  is the left side of the inequality and  $y_2$  is the right side.

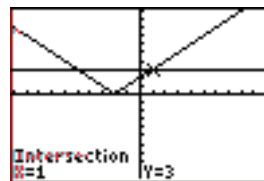
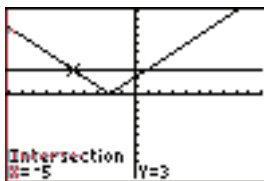
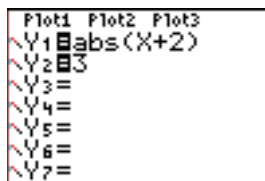


**Example** Solve  $|x + 2| > 3$  using a graphing calculator.

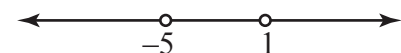
$y_1 = |x + 2|$   $y_2 = 3$

Calculator screen showing the input of the functions.

Calculator screen showing the INTERSECT function being used.



Calculator screen showing the intersection points.



Calculator screen showing the input of the functions.

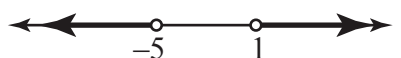
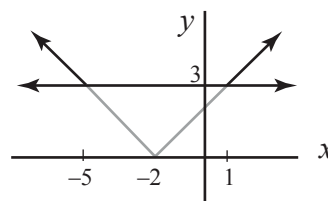
Calculator screen showing the input of the functions.

Calculator screen showing the input of the functions.

Calculator screen showing the input of the functions.

Calculator screen showing the input of the functions.

Calculator screen showing the input of the functions.



interval notation:  $(-\infty, -5) \cup (1, \infty)$

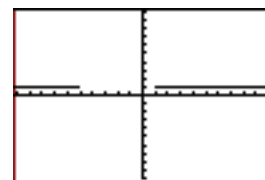
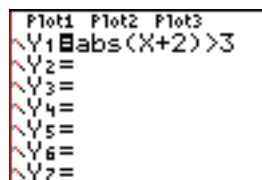
inequality:  $x < -5$  or  $x > 1$

You can also draw a number line showing this solution on your calculator with an inequality.

The inequality symbols are on the TI-84/83 Plus under  $\boxed{\boxed{\text{2ND}} \boxed{\text{<}}}$  above the  $\boxed{\boxed{\text{2ND}} \boxed{\text{>}}}$  key.

A value of “one” is plotted when the inequality is true.

A value of “zero” is plotted when it is false.



**WARNING:** This second method gives a nice picture of the solution.

But you cannot use this method to find the **endpoint values** of the interval.

Using SOOPER

You can use the equation solver on the TI-84/83 Plus to solve equations. SOLVER is available as option zero under **MATH**, in the MATH menu. Scroll down to see it.

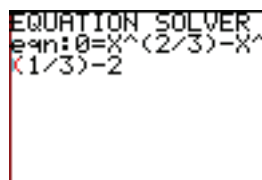
It finds approximate real solutions to equations, one at a time, using a given domain for  $x$ .

**Example 7** Use the **solver command** on your calculator to **solve**  $x^{\frac{2}{3}} = x^{\frac{1}{3}} + 2$   
Round your answer to three decimal places.

First rewrite it in the form  $f(x) = 0$ .  $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$

Press **MATH** 0 (zero, for Solver).

(If you need to, move the cursor to select the equation.)

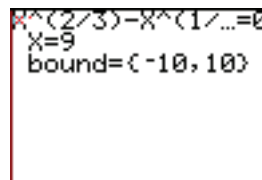



Type the equation. Press **ENTER**.

Move the cursor to bound =.

There's an  $x$ -interval given in set braces.

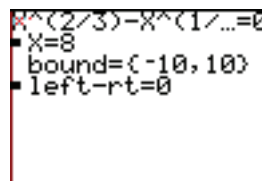
Edit this to read  $(-10,10)$ .



Move up to  $x =$  and make a guess

for  $x$  in the new interval. Let's guess 9.

So type in 9.



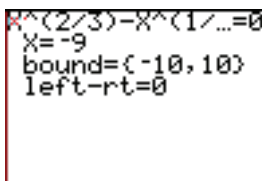
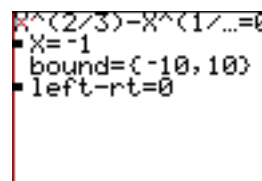
Then press **ALPHA** SOLVE (above **ENTER**).

The solution  $x = 8.000$  appears.

The left-rt at the bottom of the screen is an indication of the accuracy of the solution – the closer to zero the better.

Change the guess to  $-9$ .

Press **ALPHA** SOLVE to get the second solution.

So  $x = 8.000$  and  $x = -1.000$ .

**WARNING:** The challenge when using SOLVER is to be sure that you have found **all** of the solutions to your equation. Be sure to carefully choose your guesses and intervals for  $x$ .



Changing the Mode of the Solution

If you want to change how the solution is displayed, press **MO****□E**.

Select REAL to display only the real solutions.

Select  $a + bi$  to display complex (and real) solutions.

You can also change from normal mode

to scientific notation (**S**).

Or change to a selected number of decimal points rather than a floating-point display (**□to□t**).

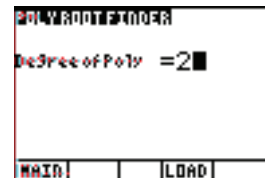


Press **□=** (for escape **E**□) to return to the last screen.

**Example** □ Solve  $x^2 + 2x = 15$  using the Polynomial Root Finder application.

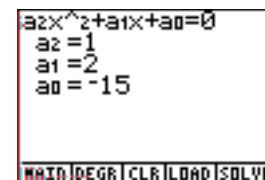
First isolate zero.  $x^2 + 2x - 15 = 0$

Then press **APPS**, select **Pol****□S****□It**, and press **ENTER**.



Press **□** (the one key for **Pol****□Root****□nder**).

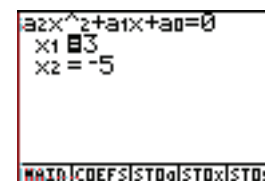
Type **□** for a degree of **□** and press **ENTER**.



Type the coefficients of the polynomial,

pressing **ENTER** after each one.

Press **□RAPH** for **SOLVE**.



The solutions are  $x = 3$  or  $x = -5$ .

**Example** □ Solve  $x^3 + 3x^2 + 3x + 1 = 0$  using the Polynomial Root Finder application.

To change the degree of the polynomial, press  $\boxed{\text{N}}\boxed{\text{O}}$  (for  $\boxed{\text{O}}\boxed{\text{E}}\boxed{\text{S}}$  or coefficients) and then  $\boxed{\text{N}}\boxed{\text{O}}$  (for  $\boxed{\text{E}}\boxed{\text{R}}$  or degree).

Type  $\boxed{3}$  (for the degree) and press  $\boxed{\text{ENTER}}$ .

Type the coefficients.

Press  $\boxed{\text{RAPH}}$  (for SOLVE).

□ait for the calculator.

□e know that  $x^3 + 3x^2 + 3x + 1 = (x + 1)^3$ .

So the three solutions are the triple solution  $x = -1$ .

□ut this calculator uses approximation techniques, giving these solutions □

$$x_1 = -0.9999396494$$

$$x_2 = -1.000030175 + 5.226869759E - 5i$$

$$x_3 = -1.000030175 - 5.226869759E - 5i$$

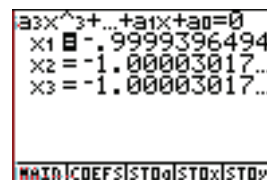
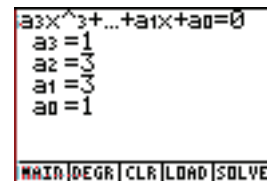
To see all of the second and third solutions scroll down and then to the right.

Two of these seem to be **imaginary** solutions.

□ut if you round them all three solutions round to  $-1 \pm 0i$ , the correct triple **real** solution of  $-1$ .

The solution is the triple root of  $x = -1$ .

**WARNING:** Be careful when you use this application to solve polynomial equations. Be sure that you interpret the solutions correctly.



## 7 Solving Systems of Equations Using a Graphing Calculator

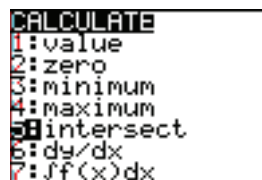
### Solving Systems of Equations Using `INTERSECT`

The intersect command on the TI-84/83 Plus finds the intersection of two curves. We can use it to solve systems of equations that have two equations and two unknowns, if the equation can be written as a function of  $x$ :  $y = f(x)$ .

**WARNING:** Always be sure that you have graphed the two functions in an appropriate window, so that you can **see all** of the intersections.

To find the intersect command:

Press `2ND` `AL` (calculate, above `TRACE`)  
option `5` (intersect)

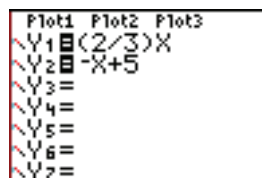


**Example** `□` Solve by graphing. 
$$\begin{cases} x = 3y \\ x + y = 5 \end{cases}$$

First rewrite the equations in the form  $f(x) = 0$ .

$$2x = 3y \Rightarrow y = \frac{2}{3}x$$

$$x + y = 5 \Rightarrow y = -x + 5$$

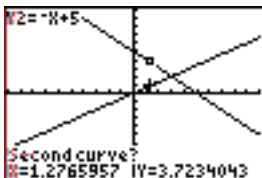


Type `□□□□` functions in the  $y=$  screen.

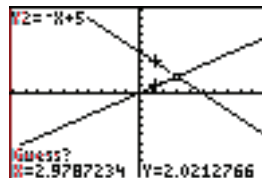
Then graph them in a standard window.

Find the intersect command as described above, and select it.

When asked for the first curve, press `ENTER` to select  $y_1$ .



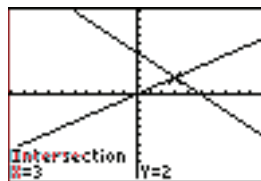
Then press `ENTER` to select  $y_2$  as the second curve.



For your guess, move the cursor close to where the cubic curve crosses the  $x$ -axis.

Then press **ENTER**.

The ordered pair solution appears at the bottom of the screen.



The solution is  $(3, 2)$ .

Notice that this is the ONLY solution, because the lines intersect in exactly one place.

**WARNING:** Be sure that it says “ $\square\square\square\square\square\square\square\square$ ”

If it says “ $\square\square\square\square$ ” then you have not yet found the solution.

**Example 2** Solve using your calculator.  $\begin{cases} x^2 + y^2 = 9 \\ y = x + 2 \end{cases}$

Round your answer to two decimal places.

First rewrite each equation in  $y = f(x)$  form.

$$x^2 + y^2 = 9 \Rightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9 - x^2} \quad \text{This is a circle.}$$

To graph it, we need  $y_1 = \sqrt{9 - x^2}$  and  $y_2 = -\sqrt{9 - x^2} = -y_1$

See Section 4, Example 4 if you need help with this.

Let  $y_3 = x + 2$ .

Graph these three functions in a Zoom-decimal window.

We need to find where  $y_3$  the line

crosses either  $y_1$  or  $y_2$  the circle

Let's find the intersection on the left.

This is the intersection of the line  $y_3$

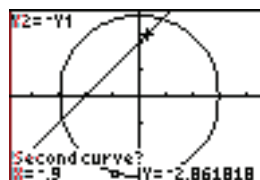
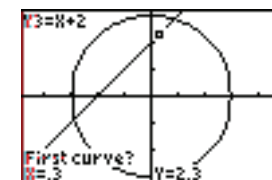
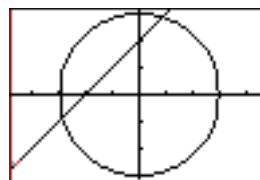
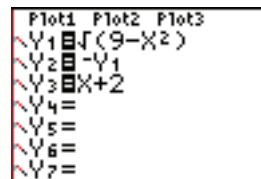
and the bottom half of the circle  $y_2$

Find and select the intersect command.

When asked for the first curve, press the up or down cursor arrow until the line is selected.

Then press **ENTER**.

When asked for the second curve, press the up or down



cursor arrow until the bottom half of the circle is selected.

Then press **ENTER**.

For your guess, move to the intersection with the right or left cursor arrow and press **ENTER**.

The solution is  $(-2.87, -0.87)$ .



Repeat this for the other solution.

This time find the intersection of  $y_1$  and  $y_3$ , the line and the top half of the circle.

The other solution is  $(0.87, 2.87)$ .

**WARNING:** Be sure that it says “**Intersection**”.  
If it still says “**Point**”, then you have not found the solution.

**NOTE:** These are approximate solutions. You can use **2nd** **MEMO** to find exact solutions to this system of equations.

### Using Your Graphing Calculator to Solve a System of Inequalities

Your calculator can help you find the solutions to a system of inequalities.

But you need to make the decision about whether to shade above or below a curve.

You must be able to write all of your curves in  $y =$  form (function of  $x$  form).

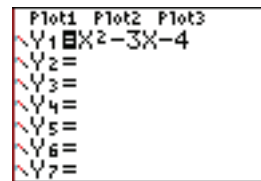
You must remember to draw the curve dotted when necessary.

**Example** Use your calculator to sketch the graph of  $y < x^2 - 3x - 4$ .

Press **Y=** and type  $x^2 - 3x - 4$  for  $y_1$ .

Notice that we need to shade **below** this curve.

Move your cursor to the left of  $y_1$  in the  $y =$  window.





press **ENTER** 3 times  
until the downward  
triangle symbol appears.

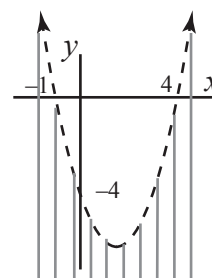
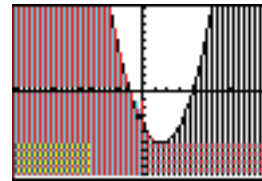
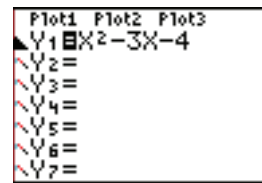
This symbol tells your calculator to  
shade **below** the curve.

The upward triangle tells your calculator  
to shade **above** the curve.

press **□□□□** **□** to graph this in a  
standard window.

sketch the solutions, using a **dotted** parabola.

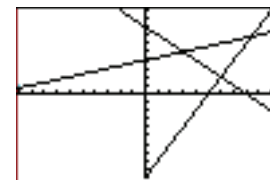
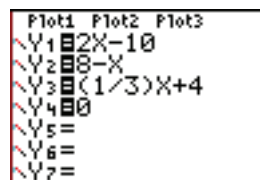
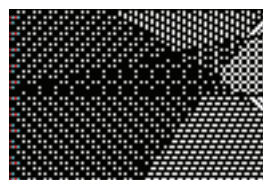
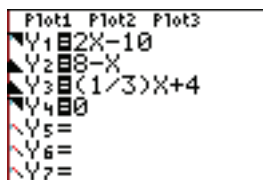
although the calculator does not use a  
dotted curve, we need to because the  
inequality did **not** include the ordered  
pairs that make  $y = x^2 - 3x - 4$ .



**Example** Sketch the graph of

$$\begin{cases} y \geq 2x - 10 \\ x + y \leq 8 \\ y \leq \frac{1}{3}x + 4 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

As powerful as the TI-84 Plus is, sometimes it is not the best way to graph an inequality. In this case, we cannot graph  $x \geq 0$ , because we cannot write  $x = 0$  in the form  $y = f(x)$ . Also, when we graph the other three inequalities using the calculator, it becomes difficult to read. It is actually easier to use the calculator to graph the lines without shading, use **INTERSECT** to find the intersections of the lines, and then shade on paper.



Use a **solid** line for each line.

Let's number the equations so that we can keep track of them.

shade **above**  $y = 2x - 10$ . Equation (1)

shade **below** and to the **left** of  $x + y = 8$ . Equation (2)

shade **below**  $y = \frac{1}{3}x + 4$ . Equation (3)

shade to the **right** of the  $y$ -axis.  $x = 0$  Equation (4)

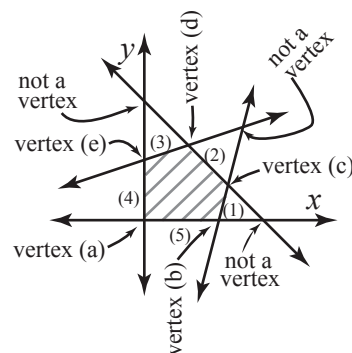
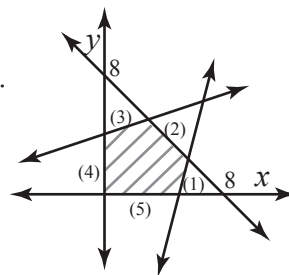
shade **above** the  $x$ -axis.  $y = 0$  Equation (5)

These five lines intersect at **ten** points.

The five that do **NOT** solve the system of inequalities are **NOT** vertices of the solutions.

Use **NOT** to find the coordinates of vertices **c**, **d** and **e**.

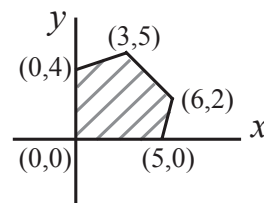
Use substitution to find vertices **a** and **b**.



vertex	(a)	(b)	(c)	(d)	(e)
the equations	(4) & (5)	(1) & (5)	(1) & (2)	(2) & (3)	(3) & (4)
the system of equations	$\begin{cases} x = 0 \\ y = 0 \end{cases}$	$\begin{cases} y = 2x - 10 \\ y = 0 \end{cases}$	$\begin{cases} y = 2x - 10 \\ x + y = 8 \end{cases}$	$\begin{cases} x + y = 8 \\ y = \frac{1}{3}x + 4 \end{cases}$	$\begin{cases} y = \frac{1}{3}x + 4 \\ x = 0 \end{cases}$
coordinates	(0,0)	(5,0)	(6,2)	(3,5)	(0,4)

label each vertex with its coordinates.

The solutions to this system of inequalities are all of the ordered pairs in the shaded region or on the line segments surrounding the region. The labeled vertices are also solutions.



### Solving a System of Linear Equations Using $\square\square\square\square\square\square$ $\square$ T

$\square\square\square\square\square$  is an application program available for your TI-84/83 Plus.

You can download it from the Texas Instruments Web site

(<http://education.ti.com/educationportal/sites/US/sectionHome/download.html>)

or transfer it from another TI-84/83 Plus calculator if it did not come preloaded on your calculator.

$\square\square\square\square\square$  (Polynomial Root Finder and Simultaneous Equation Solver) can solve systems of linear equations. (It also finds real and complex solutions to polynomial equations. See Section 6.)

### Running the Polynomial Root Finder and Simultaneous Equation Solver

To run this application, press  $\square\square\square\square$ .

select  $\square\square\square\square\square$  and press  $\square$ ENTER $\square$ .

To exit the application, press  $\square$  in the main menu *for*  $\square$ uit  $\square$ oly $\square$ mlt $\square$  or press  $\square$   $\square$ T.



### The Simultaneous Equation Solver

This part of Polynomial Root Finder and Simultaneous Equation Solver application solves linear systems of equations.

### Solving a Linear System of Equations

This application uses Gauss-Jordan elimination to solve linear systems of equations. Start by rewriting the linear equations with the variables in order on the left side of each equation and the constants on the right side.

From the main menu of  $\square\square\square\square\square$   $\square$ , press  $\square$   $\square$   $\square$   $\square$ E $\square\square\square\square\square\square$ .

Type the number of equations and the number of unknowns (variables), pressing  $\square$ ENTER $\square$  after each one.

Type the coefficients and constants from your system of equations pressing  $\square$ ENTER $\square$  after each one.

Press  $\square$ R $\square\square\square$  *for*  $\square\square\square$ E $\square$  to see the solution.

Press  $\square$ TR $\square\square$ E $\square$  *for* RRE $\square$  to see the matrix in reduced row echelon form.



**Example**  $\square$  Solve 
$$\begin{cases} x + 3y - 7 = 0 \\ x = 4y + 2 \end{cases}$$
 using the Simultaneous Equation Solver application.

Rewrite the equations with the variables, in order, on the left side and the constants on the right side.

$$\begin{cases} x + 3y = 7 \\ x - 4y = 2 \end{cases}$$

Press  $\square\square\square\square$ , select  $\square\square\square\square\square$  and press  $\square$ ENTER.

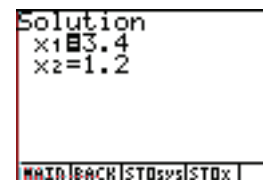
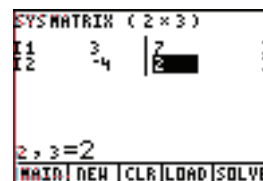
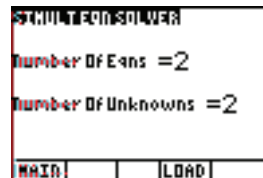
Press  $\square$  for  $\square\square\square\square$  E  $\square\square\square\square\square\square\square$ .

Type  $\square$  for two equations  $\square$ ENTER and then  $\square$  for two variables  $\square$ ENTER.

Type the coefficients and constants from the system, pressing  $\square$ ENTER after each one.

Press  $\square$ R  $\square\square\square\square$  for  $\square\square\square\square$  E and wait for the calculator to complete its calculations.

The solution is (3.4, 1.2).



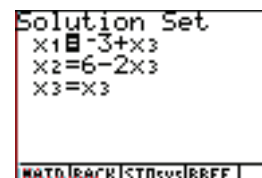
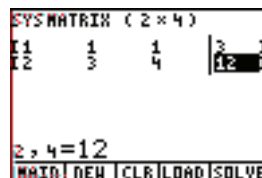
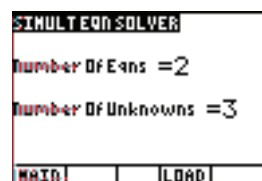
**Example**  $\square$  Solve 
$$\begin{cases} x + y + z = 3 \\ x + 3y + 4z = 12 \end{cases}$$
 using the Simultaneous Equation Solver application.

Press  $\square$ IN  $\square\square\square\square$  for  $\square\square\square\square$  twice to return to the screen showing the number of equations.

Press  $\square$   $\square$ ENTER  $\square$   $\square$ ENTER.

Type the coefficients and constants from the system, pressing  $\square$ ENTER after each one.

Press  $\square$ R  $\square\square\square\square$  for  $\square\square\square\square$  E to see the solution.



The solution, in ordered-triple notation, is  $(-3 + t, 6 - 2t, t)$ .

Or we can write the solution as 
$$\begin{cases} x = \square - 3 \\ y = 6 - 2\square \\ \square = \square \end{cases}$$

If you are interested in how your calculator can solve linear systems of equations using Gaussian Elimination, Gauss-Jordan Elimination or Cramer's Rule, see Section 8.



**Example 2** Edit matrix  $A$  to read  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$ .

Press  $\boxed{\text{2nd}} \boxed{\text{MATRX}}$  and move to the right to select  $E \rightarrow T$ .

Then press  $\boxed{\text{2nd}} \boxed{\text{F1}}$  to select  $A$ .

Then move the cursor down and to the right so that it is on top of the number  $10$ .



Replace it by typing  $10$  and then press  $\boxed{\text{ENTER}}$ .

WARNING: The entry is  $10$  changed until you press  $\boxed{\text{ENTER}}$ .

Press  $\boxed{\text{2nd}} \boxed{\text{QUIT}}$  to return to the home screen.

### Elementary Row Operations

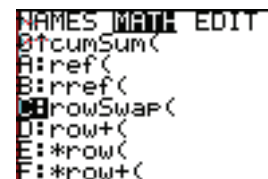
We can use the TI-84/83 Plus to do the three elementary row operations:

1. interchange two rows. (Swap rows.)
2. multiply a row by a non-zero constant.
3. add a constant multiple of one row to the corresponding entries of another row.

NOTE: See Examples 1 and 2 for how to enter and edit a matrix.

### Swapping Rows

To interchange or swap two rows, press  $\boxed{\text{2nd}} \boxed{\text{MATRX}}$  and move to the right to select MATH.

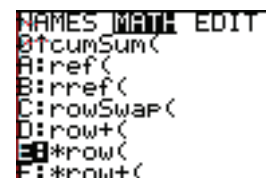


Then select option C for rowSwap(.

The format for this option is rowSwap(the matrix, the first row, the second row).

### Multiplying a Row by a Constant

To multiply a row by a constant, press  $\boxed{\text{2nd}} \boxed{\text{MATRX}}$  and move to the right to select MATH.



Then select option E for \*row(.

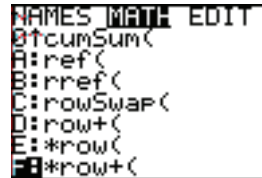
The format for this option is \*row (the constant, the matrix, the row).

### Adding a Multiple of a Row to Another Row

To add a multiple of a row to another row,

press  $2^{nd}$  MATRIX

and move to the right to select MATH.



Then select option  $\square$  for  $*row\square($ .

The format for this option is

$*row +$ (the constant, the matrix, the row that is multiplied, the row that is added to).

**Example 3** Using matrix  $\square = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$ , find the new matrix when

- rows 1 and 3 are interchanged
- row 1 is multiplied by 3
- row 1 is multiplied by  $-2$  and added to row 2

a) rows 1 and 3 are interchanged

$\square$ nter m $\square$ tr $\square\square\square$  into  $\square$ our c $\square$ cu $\square\square$ tor.  $\square$ ee  $\square\square$ m $\square$ e  $\square$

Press  $2^{nd}$  QUIT to return to the home screen.

Press  $2^{nd}$  MATRIX

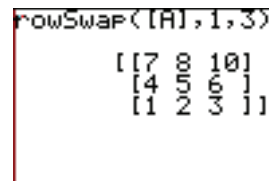
$\square$ n $\square$ mo $\square$ e to the r $\square$ ht to se $\square$ ct MATH.

$\square$ hen  $\square$ ress  $\square\square\square\square$  C  $\square$ or rowSwap(.

Press  $2^{nd}$  MATRIX  $\square$   $\square$ or m $\square$ tr $\square\square\square$

$\square$  the comm $\square$ e $\square\square$ s  $\square$ o $\square$ e the  $\square$ e $\square\square$   $\square$ or ro $\square$  one $\square$

$\square$   $\square$ or ro $\square$  three $\square$   $\square$ n $\square$   $\square$ ENTER $\square$ .

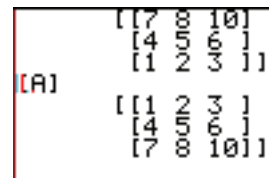


Notice that the new matrix appears on the calculator screen.

BUT matrix  $\square$ , as stored, does not change.

$\square$ o  $\square$ er $\square\square$ th $\square$   $\square$ ress  $2^{nd}$  MATRIX

$\square$   $\square$ or m $\square$ tr $\square\square\square$ n $\square$   $\square$ ENTER $\square$ .





You can store the newly created matrix as another matrix.

To do so

☐ress ☐2<sup>nd</sup> ENTR ☐unt ☐

rowSwap(  errors on the screen.

Press **ENTER** to return the ne m tr to the home screen.

When Press **ST**  **2<sup>nd</sup>** **MATRX**

2  or  $m \times n$   ENTER.

Your new matrix is now stored as matrix .

If desired, you can store your new matrix as matrix  $\square$ , but then you would no longer have the original matrix  $\square$  stored in your calculator.

```

[4 5 6]
[7 8 10]]
rowSwap([A],1,3)

[7 8 10]
[4 5 6]
[1 2 3]]

```

$$\text{Ans} \rightarrow [B] = \begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

b) row 1 is multiplied by 3

Comutatorress  $2^{nd}$  MATRIX ☐ move to the right to select MATH.

When Press **A** **H** **A** **E** for \*row(.

[illegible]

2<sup>nd</sup> MATRX ☐ or m|  |  |  |  |

☐ ☐ or *or* ☐ *one* ☐ *n* .

```
*row(3,[A],1)
[3 6 9 ]
[4 5 6 ]
[7 8 10]
```

c) row 1 is multiplied by  $-2$  and added to row 2

to multiply  $n$  then the result to  $2^{\text{nd}}$  MATRIX

☐ *move to the right to select MATH.*

then press **A⇐HA** for \*row(.

*Press* (-) [2] or mult [ ] n

2<sup>nd</sup> MATRX ☐ or m|  |  |  |  |

☐ ☐ ☐ *or* *ro* ☐ *one* ☐

☐ 2 or more tokens ENTER.

```
*row+(-2,[A],1,2)
```

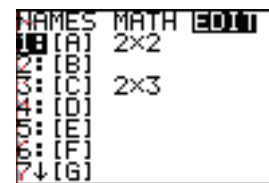
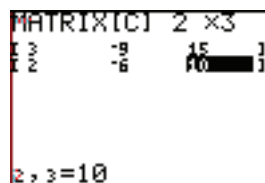
### Using Your Calculator to Do Matrix Operations

You can use the TI-84/83 Plus to do matrix operations. To do so, enter the matrices and then do the operations on the home screen of the calculator.

**Example 4** If  $A = \begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -9 & 15 \\ 2 & -6 & 10 \end{bmatrix}$

use your graphing calculator to find a)  $AB$  b)  $BA$  and c)  $A^T B$

First enter the matrices in your calculator. See the example.



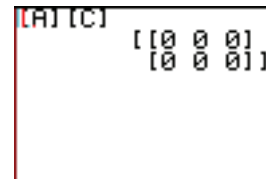
Press  $\boxed{2^{nd}}$  QUIT to return to the home screen.

a) Find  $AB$ .

Press  $\boxed{2^{nd}}$  MATRIX  $\boxed{A}$  or  $m\boxed{tr}\boxed{1}$ .

Then press  $\boxed{2^{nd}}$  MATRIX  $\boxed{B}$  or  $m\boxed{tr}\boxed{2}$   $\boxed{ENTER}$ .

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



b) Find  $BA$ .

Press  $\boxed{2^{nd}}$  MATRIX  $\boxed{B}$  or  $m\boxed{tr}\boxed{2}$ .

Then press  $\boxed{2^{nd}}$  MATRIX  $\boxed{A}$  or  $m\boxed{tr}\boxed{1}$   $\boxed{ENTER}$ .

The error message tells us that this product is undefined.

The dimensions of the matrices do not match.

Press  $\boxed{ENTER}$  to return to the home screen.

$BA$  is undefined.



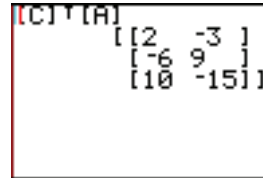
c) Find  $A^T$ .

Press  $2^{nd}$  MATRIX  $\square$   $2^{nd}$  MATRIX.

Go to the right to select MATH.

Then press  $2$  for transpose

$2^{nd}$  MATRIX  $\square$  ENTER.



$$\begin{bmatrix} 2 & -3 \\ -6 & 9 \\ 10 & -15 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & -3 \\ -6 & 9 \\ 10 & -15 \end{bmatrix}$$

### Using Your Graphing Calculator to Find an Inverse Matrix

Using your calculator is the easiest way to find an inverse matrix. Just be careful that you do not approximate the matrix.

**Example 5** Use your graphing calculator to find  $A^{-1}$  if

a)  $A = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}$       b)  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

a) Find  $A^{-1}$  if  $A = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}$

Enter the matrix in your calculator.

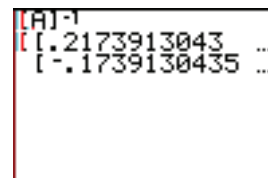
See Example 4

Press  $2^{nd}$  QUIT to return to the home screen.

Press  $2^{nd}$  MATRIX 1 to select matrix.

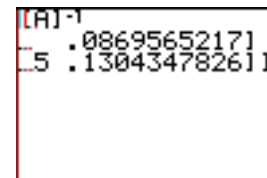
Press  $\square^{-1}$  ENTER.

Press the right cursor arrow to see the rest of the matrix.



$$\begin{bmatrix} .2173913043 & \dots \\ -.1739130435 & \dots \end{bmatrix}$$

Round the value  $A^{-1} = \begin{bmatrix} 0.22 & 0.09 \\ -0.17 & 0.13 \end{bmatrix}$



$$\begin{bmatrix} .0869565217 & \dots \\ .1304347826 & \dots \end{bmatrix}$$

Let's check this.

Take the values from our rounded  $\square^{-1}$  into matrix.

Then multiply them on our calculator.

Notice that none of these is equal to the identity matrix.

They're not even equal to each other.

So a **rounded form** for  $\square^{-1}$  is **NOT** the same as  $\square^{-1}$ .

WARNING: A decimal approximation of  $\square^{-1}$  is **NOT** the same as  $\square^{-1}$ .

If you must use a decimal form, use as many digits as possible to increase your accuracy.

Let's get in exact form of this inverse matrix.

Press  $2^{nd}$  ENTR until  $\square^{-1}$  appears in the display line of our calculator.

Then press **MATH**  $\square$  for the convert to fraction function.

Then press **ENTER**.

$$\text{So } \square^{-1} = \begin{bmatrix} \frac{5}{23} & \frac{2}{23} \\ -\frac{4}{23} & \frac{3}{23} \end{bmatrix}$$

To check on our calculator

store these new matrices in our calculator

Press  $\square$  **STO**  $\rightarrow$   $2^{nd}$  **MATRX**  $\square$  to select matrix.

So multiply them.

Notice that we now get the  $2 \times 2$  identity matrix.

MATRIX(B) 2 x 2  
 $\begin{bmatrix} .22 & .09 \\ .17 & .13 \end{bmatrix}$   
 $z, z = .13$

[A][B]  
 $\begin{bmatrix} 1 & .01 \\ .03 & 1.01 \end{bmatrix}$   
[B][A]  
 $\begin{bmatrix} 1.02 & .01 \\ .01 & .99 \end{bmatrix}$

[B][A]  
 $\begin{bmatrix} .03 & 1.01 \\ 1.02 & .01 \\ .01 & .99 \end{bmatrix}$   
[A]  $\rightarrow$  Frac  
 $\begin{bmatrix} 5/23 & 2/23 \\ -4/23 & 3/23 \end{bmatrix}$

[A]  $\rightarrow$  Frac  
 $\begin{bmatrix} 5/23 & 2/23 \\ -4/23 & 3/23 \end{bmatrix}$   
Ans  $\rightarrow$  [B]  
 $\begin{bmatrix} .2173913043 & ... \\ -.1739130435 & ... \end{bmatrix}$

[A][B]  
 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   
[B][A]  
 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) Find  $A^{-1}$  if  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Repeat the procedure from part (a).

The error message tells us that  $A$  is singular matrix.

Press **ENTER** to return to the home screen.

So  $A$  does NOT have an inverse matrix.

### Using Matrix Algebra to Solve a System of Linear Equations

We can use matrix multiplication to represent a system of linear equations.

The system of equations  $\begin{cases} 2x + y - 3z = -1 \\ 7x + 2y + z = 1 \\ 3x - y + 2z = 6 \end{cases}$  can be written  $\begin{bmatrix} 2 & 1 & -3 \\ 7 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$

To see this, multiply the matrices on the left side of the equation.  $\begin{bmatrix} 2x + y - 3z \\ 7x + 2y + z \\ 3x - y + 2z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$

Then set corresponding entries equal.

If we let  $A = \begin{bmatrix} 2 & 1 & -3 \\ 7 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  and  $B = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$ ,

then we have the matrix equation  $AX = B$ .

If we solve this matrix equation for  $X$ , then we solve the system of linear equations.

To find matrix  $\mathbf{X}$ , multiply both sides of the matrix equation by  $\mathbf{A}^{-1}$ .

This gives us  $\mathbf{A}^{-1}\mathbf{A}\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ .

Simplifying, we have  $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ .

This gives us the solution to the system of linear equations.

**WARNING:** Remember that the order of multiplication is very important when you are multiplying matrices.  $\mathbf{A}^{-1}$  is equal to  $\mathbf{A}^{-1}\mathbf{A}$ . Do **NOT** find  $\mathbf{A}\mathbf{A}^{-1}$ , this is a different product.

**WARNING:** This procedure will **NOT** work if  $\mathbf{A}$  is a singular matrix. When  $\mathbf{A}$  is singular, it is better to use substitution, elimination or Gauss-Jordan elimination to solve the system of equations.

**Example 6** Use matrix algebra to solve 
$$\begin{cases} 2x + y - 3z = -1 \\ 7x + 2y + z = 1 \\ 3x - y + 2z = 6 \end{cases}$$

$$\text{Here } \mathbf{A} = \begin{bmatrix} 2 & 1 & -3 \\ 7 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}.$$

We need to find  $\mathbf{A}^{-1}\mathbf{B}$ .

Use the  $\text{t}\text{o}\text{m}\text{tr}\text{ces}$   $\text{on}$  then  $\text{on}$   $\mathbf{A}^{-1}\mathbf{B}$ .

See  $\text{m}\text{es}$   $\text{on}$ .

Press  $\text{2nd}$  QUIT to return to the home screen.

$$\text{So we have that } \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

$$\text{MATRIX[A] } 3 \times 3$$

$$\begin{bmatrix} 2 & 1 & -3 \\ 7 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix}$$

$$\text{MATRIX[B] } 3 \times 1$$

$$\begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$$

$$[\text{A}]^{-1}[\text{B}]$$

$$\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

Let  $x$  correspond to the entries  $e$   $u$   $e$   $h$   $e$   $x = 1$ ,  $y = -3$  and  $z = 0$ .

The solution is  $(1, -3, 0)$ .

**WARNING:** If matrix  $\mathbf{A}$  has a decimal form, and you need a rational answer, remember to use the “convert to fraction” function available under the MATH key of your calculator.



$$\text{This gives us the new system of equations } \begin{cases} x - \frac{1}{2}y + \frac{5}{6}z = \frac{2}{3} \\ x - \frac{17}{9}y = -\frac{7}{9} \\ z = \frac{1}{2} \end{cases}$$

Use the  $\rightarrow$  key to solve this system of equations.

The exact solution is  $x = \frac{1}{3}$ ,  $y = \frac{1}{6}$ ,  $z = \frac{1}{2}$ .

It is easier to solve this system on our calculator using our  $\rightarrow$  key function. See the next example.

### Using Your Calculator to Do Gauss-Jordan Elimination

The TI-84/83 Plus can quickly reduce an augmented matrix to its reduced row-echelon form, allowing us to solve systems of equations using Gauss-Jordan elimination.

**Example 8** Solve using Gauss-Jordan elimination and your calculator.

$$\begin{cases} x + y + 3z = 2 \\ 6x - 3y + 5z = 4 \\ 2x + 2y - 4z = -1 \end{cases}$$

First form the augmented matrix for this system.

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 6 & -3 & 5 & 4 \\ 2 & 2 & -4 & -1 \end{array} \right]$$

Use the  $\rightarrow$  key to solve this system.

Use the cursor  $\rightarrow$  key to check our entries in the matrix.

MATRIX[A] 3x4			
1	1	3	2
6	-3	5	4
2	2	-4	-1

MATRIX[A] 3x4			
1	1	3	2
6	-3	5	4
2	2	-4	-1

Press  $2^{nd}$  QUIT to return to the home screen.



Press  $2^{nd}$  MATRX  $\rightarrow$  move to MATH.

Then press  $\boxed{A}\boxed{+}\boxed{HA}$  for  $rref()$ .

Press  $2^{nd}$  MATRX  $\rightarrow$   $\rightarrow$  or  $\rightarrow$ ) ENTER

Now you can see the row echelon form  $(0.333, 0.167, 0.500)$ .

Now the exact solution

press  $2^{nd}$  ENTR  $\rightarrow$  MATH  $\rightarrow$  or  $\rightarrow$   $\rightarrow$  ENTER

```

NAME: MATH EDIT
8:Matr→list(
9:List→matr(
0:cumSum(
A:rref(
B:rref(
C:rowSwap(
D:row+(

```

```

rref([A])
[[1 0 0 .333333...
[0 1 0 .166666...
[0 0 1 .5 ...

```

```

[[1 0 0 .333333...
[0 1 0 .166666...
[0 0 1 .5 ...
rref([A])→Frac
[[1 0 0 1/3]
[0 1 0 1/6]
[0 0 1 1/2]

```

And we have the exact solution  $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$ .

If you're careful, you can also use this method to solve inconsistent, dependent or non-square linear systems of equations.

### Using Your Calculator to Do Cramer's Rule

We can use the TI-84/83 Plus to evaluate determinants. So we can solve systems of equations using Cramer's Rule on the calculator. Your calculator is especially useful when the numbers are messy, or when the system is larger than three equations, three unknowns.

**Example 9** Solve using Cramer's rule and your calculator.

$$\begin{cases} x + y + 3z = 2 \\ 6x - 3y + 5z = 4 \\ 2x + 2y - 4z = -1 \end{cases}$$

First form the four determinants used for Cramer's rule.

$$D = \begin{vmatrix} 1 & 1 & 3 \\ 6 & -3 & 5 \\ 2 & 2 & -4 \end{vmatrix} \quad D_x = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -3 & 5 \\ -1 & 2 & -4 \end{vmatrix} \quad D_y = \begin{vmatrix} 1 & 2 & 3 \\ 6 & 4 & 5 \\ 2 & -1 & -4 \end{vmatrix} \quad D_z = \begin{vmatrix} 1 & 1 & 2 \\ 6 & -3 & 4 \\ 2 & 2 & -1 \end{vmatrix}$$

Be sure to use the correct signs for the minors in the determinant.

Let  $D = D$ ,  $A = D_x$ ,  $B = D_y$  and  $C = D_z$ .

Press  $2^{nd}$  MATRIX  $\rightarrow$  move  $\boxed{C^{-1}}$   $\rightarrow$

move to the right to select  $\boxed{EDIT}$  then press  $\boxed{\rightarrow}$  to select A.

Press  $\boxed{\rightarrow}$   $\boxed{ENTER}$   $\boxed{\rightarrow}$  to move A rows and columns.

move the entries from determinant  $A = D$

be sure to press  $\boxed{ENTER}$  after each entry

Press  $2^{nd}$  QUIT to return to the home screen.

repeat this procedure to fill B, C, and D. here's our four core matrices now

Press  $2^{nd}$  QUIT to return to the home screen.

Press  $2^{nd}$  MATRIX  $\rightarrow$

move to MATH then press  $\boxed{\rightarrow}$  and/or determinant

then press  $2^{nd}$  MATRIX  $\rightarrow$  or  $\boxed{\rightarrow}$   $\boxed{\div}$

$2^{nd}$  MATRIX MATH  $\rightarrow$  or determinant

$2^{nd}$  MATRIX  $\rightarrow$  or  $\boxed{\rightarrow}$   $\boxed{\div}$

MATH  $\rightarrow$  to convert to fraction form  $\boxed{ENTER}$ .

This is  $\frac{A}{D} = \frac{1}{3}$ .

to then press  $2^{nd}$  ENTER move  $\boxed{ENTER}$   $\rightarrow$

move the cursor up and on to A the result

Press  $2^{nd}$  MATRIX  $\boxed{2}$  to replace it then press  $\boxed{ENTER}$ .

So  $\frac{A}{D} = \frac{1}{3}$ . then then to find  $\frac{A}{D} = \frac{1}{3}$ .

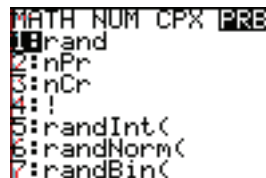
The solution is  $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$ .

## 9 Probability and Statistics

### MATH Probability Options

There are seven options under the MATH  $\square R \square$  (probability) menu.

To see this menu press  $\square MATH \square$  then move to the right to select  $\square R \square$ .



### Generating Random Numbers

The rand option (option 1) generates random numbers between 0 and 1.

The TI-84/83 Plus generates its random values based on a starting “seed” value for the rand function. The factory set seed value is 0 (zero). Reset this seed value by storing a new seed value in rand.

- Example 1** Reset the seed value for rand to 47. Then
- generate three random numbers between 0 and 1
  - generate six random numbers between 0 and 10

To reset the seed value

press  $\square \square \square STO \square MATH \square$

move to the right to select  $\square R \square$ ,

press  $\square ENTER \square$  to select rand

then press  $\square ENTER \square$  to store  $\square \square$  as the seed value



(a) generate three random numbers between 0 and 1

Here are three different ways to generate three random numbers

The first method is to press  $\square MATH \square$

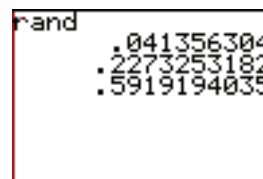
move to the right to select  $\square R \square$ ,

press  $\square ENTER \square$  to select rand

then press  $\square ENTER \square$  to get the first random number

then press  $\square ENTER \square$  two more times

to get the next two random numbers.



The second method is to include the number of random numbers you want as an argument in the rand function.

To do this, press **MATH**, move to the right to select PRB, press **ENTER** (to select rand) then press **( ) 3 ( )** (for 3 random numbers).

Use the right cursor arrow to see the three random numbers in the list.

```
rand
.041356304
.2273253182
.5919194035
rand(3)
.0537687915 .2...
```

```
rand
.041356304
.2273253182
.5919194035
rand(3)
.5 .2744389575 ...
```

```
rand
.041356304
.2273253182
.5919194035
rand(3)
.575 .991431377)
```

NOTE: Notice that these new numbers are different from the previous three. If you reset the seed value to 47 again, and then run rand(3) you will get the **first** three random numbers  $\{0.041356304, 0.2273253182, 0.5919194035\}$  again.

(b) generate six random numbers between 0 and 10

We need numbers 10 times as large as the numbers in part (a).

So press **1 0 \* MATH**,

move to the right to select PRB,

press **ENTER** (to select rand)

and press **ENTER** (to get the first random number).

Continue to press **ENTER** until there are six numbers on the screen.

```
10*rand
.5376879155
2.744389575
9.91431377
7.629633004
2.296712709
9.791381216
```

### Combinations and Permutations

We can find the number of permutations,  ${}_nP_r$ , or combinations,  ${}_nC_r$  or  $\binom{n}{r}$ , using options 2 and 3 in the PRB menu.

Be sure to enter the value for  $n$  on the home screen before accessing the MATH PRB menu.

□□□□ □□□□ Use your calculator to find (a)  ${}_{10}P_4$  (b)  ${}_{10}C_4$  (c)  $\binom{100}{2}$

(a)  ${}_{10}P_4 = 5040$

Press **1** **0** **MATH**, move to the right to select PRB, press **□** (option **□** for  ${}_nP_r$ ),  
**□** **ENTER**.

(b)  ${}_{10}C_4 = 210$

Press **1** **0** **MATH**, move to the right to select PRB, press **3** (option 3 for  ${}_nC_r$ ),  
**□** **ENTER**.

(c)  $\binom{100}{2} = {}_{100}C_2 = 4950$

Press **1** **0** **0** **MATH**, move to the right to select PRB, press **3** (option 3),  
**□** **ENTER**.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
10 nPr 4      5040
10 nCr 4      210
100 nCr 2     4950
```

### Factorial

The fourth option in the MATH PRB menu is factorial.

Be sure to type the  $n$  in  $n!$  before accessing the MATH PRB menu.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

□□□□ □□□□ Use your calculator to find (a)  $10!$  (b)  $100!$

(a)  $10!$

Press **1** **0** **MATH**, move to the right to select PRB,  
press **□** (option **□** for factorial), **ENTER**.

```
10!      3628800
```

(b)  $100!$

Press  $\boxed{1}\boxed{0}\boxed{0}$   $\boxed{\text{MATH}}$ , move to the right  
to select PRB, press  $\boxed{\text{2nd}}$   $\boxed{\text{option}}$   $\boxed{\text{3}}$ ,  $\boxed{\text{ENTER}}$ .

```
ERR:OVERFLOW
1:Quit
2:Goto
```

NOTE: The  $\boxed{\text{ERR:OVERFLOW}}$  error message appears when the number exceeds the range of the TI-84/83 calculator. This is happening here because  $100!$  is a very large number.

### Other Random Number Generators

The last three options in the MATH PRB menu generate other sets of random numbers. Use the following formats:

#### Option 5 - randInt(

randInt(lower integer bound, upper integer bound)

This generates one random integer within the range given by the lower and upper bounds, including the lower and upper bounds.

```
randInt(1,5)
randInt(1,5,3) 3
               (2 4 3)
```

randInt(lower integer bound, upper integer bound,  $m$ )

This generates  $m$  random integers within the range given by the lower and upper bounds.

#### Option 6 - randNorm(

randNorm( $\mu$ ,  $\sigma$ )

This generates one random real number from a normal distribution with the given mean  $\mu$  and standard deviation  $\sigma$ .

randNorm( $\mu$ ,  $\sigma$ ,  $m$ )

This generates  $m$  random real numbers from a normal distribution with the given mean  $\mu$  and standard deviation  $\sigma$ .

```
randNorm(10,2)
11.73537766
randNorm(10,2,3)
(12.09761301 10...
```

```
randNorm(10,2)
11.73537766
randNorm(10,2,3)
10.2148691 10....
```

```
randNorm(10,2)
11.73537766
randNorm(10,2,3)
91 10.86872334)
```

Option 7 – randBin(randBin( $n, \square$ )

This generates one random integer from a binomial distribution with  $n$  times a trial is repeated ( $n \geq 1$ ) and probability  $\square$  of success in a single trial ( $0 \leq \square \leq 1$ ).

```
randBin(5,0.4)
randBin(5,0.4,3)
      (2 2 3)
```

randBin( $n, \square, m$ )

This generates  $m$  random integers from a binomial distribution with  $n$  times a trial is repeated ( $n \geq 1$ ) and probability  $\square$  of success in a single trial ( $0 \leq \square \leq 1$ ).

Lists

The TI-84/83 Plus has six list names in memory. They are designated

$\square_1$ ,  $\square_2$ ,  $\square_3$ ,  $\square_4$ ,  $\square_5$  and  $\square_6$ . You can access these lists by pressing the second key and the number keys from 1 to 6.

You can view all six lists by pressing  $\square$ TAT (next to the cursor keys) and pressing  $\square$ ENTER (to select EDIT). Move the cursor to the right to see all 6 lists.

```
EDIT CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
```

L1	L2	L3	1
-----	-----	-----	
L1(1)=			

L4	L5	L6	6
-----	-----	-----	
L6(1)=			

Creating and Naming a New List

You can create additional lists with custom names, if needed. To create a new list with a new name, move the cursor up to the names of the list and to the right until you have a blank name at the top of the list.

Notice that the cursor is now in alpha mode.

Type the new name for the list and press  $\square$ ENTER.

L5	L6	NAME	?
-----	-----	-----	
Name=			

L5	L6	NAME	?
-----	-----	-----	
LIST =			

### Entering Data in a List

You can enter data in a list on the home screen of in the  $\boxed{\text{TAT}}$   $\boxed{\text{EAT}}$  option.

On the home screen, type the list in set braces with commas between each number. Then store the list as one of the list names.

In the  $\boxed{\text{TAT}}$   $\boxed{\text{EAT}}$  option, type the list pressing the  $\boxed{\text{ENTER}}$  after each number in the list.

$\boxed{\text{L1}}$   $\boxed{\text{L2}}$  Store  $\{1, 7, 3, 5\}$  as  $\boxed{\text{L1}}$  and  $\{10, 2, 13, 3, 7\}$  as  $\boxed{\text{L2}}$ .

Let's do the first list on the home screen.

Press  $\boxed{\{}$  above  $\boxed{(}$ , type the numbers in the list, separated by commas  $\boxed{,}$  above  $\boxed{}$  and then press  $\boxed{\}}$ ,  $\boxed{\text{TAT}}$   $\boxed{\text{nd}}$   $\boxed{\text{L1}}$  above  $\boxed{1}$ .

To see the list, press  $\boxed{\text{TAT}}$   $\boxed{\text{ENTER}}$  to select  $\boxed{\text{EAT}}$ .

If you need to, move the cursor to the left to see  $\boxed{\text{L1}}$ .

L1	L2	L3	1
1 7 3 5			
L1(1)=1			

We have the lists on the screen, so we will enter the second list here.

Move the cursor to the right. Type each number followed by  $\boxed{\text{ENTER}}$ .

L1	L2	L3	2
1 7 3 5	10 2 13 3 7		
L2(6)=			

**WARNING:** A number is not entered into the list until you press  $\boxed{\text{ENTER}}$ .

Be sure to press  $\boxed{\text{ENTER}}$  after you type your last entry.



Sorting a List

You can sort a list in either ascending,  $\square\square A($ , or descending,  $\square\square D($ , order.

You can find these options by pressing  $\square^{nd} \square\square T$  (above  $\square TAT$ ) and then moving to  $\square P\square$  (for operations).

$\square\square\square\square\square\square\square$  Sort  $\square_1$  in ascending order and  $\square_2$  in descending order.

Press  $\square^{nd} \square\square T$  (above  $\square TAT$ )

and then move to  $\square P\square$  (for operations).

Press  $\square ENTER$  or  $\square 1$  to select  $\square\square A($ .

Then press  $\square^{nd} \square_1$  (above  $\square 1$ )  $\square ENTER$ .

```
SortA(L1)      Done
SortD(L2)      Done
```

Press  $\square^{nd} \square\square T$  (above  $\square TAT$ )

and then moving to  $\square P\square$  (for operations).

Press  $\square$  to select  $\square\square D($ .

Then press  $\square^{nd} \square_2$  (above  $\square$ )  $\square ENTER$ .

L1	L2	L3	2
1	13		
3	10		
5	8		
7	6		

L2(1)=13

To see the lists, press  $\square TAT$

and  $\square ENTER$  to select  $\square\square T$ .

If you need to, move the cursor to the left to see  $\square_1$ .

Deleting a List

You can delete a list by moving to the name at the top of the list and pressing  $\square\square R$  (for clear) and then  $\square ENTER$ .

L1	L2	L3	1
1	13		
3	10		
5	8		
7	6		

L1={1,3,5,7}

L1	L2	L3	1
1	13		
3	10		
5	8		
7	6		

L1(1)=

Statistics – One Variable Data

The TI-84/83 Plus can quickly calculate the following for a given list of  $x$ -values:

$\square\square\square\square$ <b>2nd</b> <b>LIST</b> $\square$	$\square\square$ <b>2nd</b> <b>LIST</b> $\square\square$
The mean	$\bar{x}$
The sum of the $x$ -values	$\sum x$
The sum of the squares of the $x$ -values	$\sum x^2$
The sample standard deviation	$S_x$
The population standard deviation	$\sigma_\square$
The number of data points	$n$
The minimum of the $x$ -values	$\square$ <b>min</b> $\square$
The maximum of the $x$ -values	$\square$ <b>a</b> $\square\square$
The first quartile	$\square_1$
The median	<b>M</b> $\square$ <b>d</b>
The third quartile	$\square_3$

To get this information about a list, first type your data in a list.

Then press **2nd** **TAT** move to the right to **A**  $\square\square$ , press **ENTER** to select

1  $\square\square$  **ar**  $\square$  **a**  $\square\square\square\square\square$  (for one-variable statistics) then type the list name and press **ENTER**.

If the data is frequency data, store the frequencies in another list.

Then use the format 1  $\square\square$  **ar**  $\square$  **a**  $\square$  list of values, list of frequencies.

$\square\square\square\square$   $\square\square\square\square$  Find the mean and sample standard deviation for the twenty quiz scores in the list  $\{2, 4, 5, 6, 3, 6, 5, 4, 5, 4, 5, 7, 8, 2, 6, 7, 8, 7, 9, 9\}$

Enter the data in  $\square_1$ .

(If you already have data in  $\square_1$ , delete it first.)

Before continuing, check your typing.

Move the cursor through the list and correct any by typing on top of the incorrect value.

Be sure to press **ENTER** after the last number in the list.

L1	L2	L3	1
2			
4			
5			
6			
3			
6			
5			
4			
5			
4			
5			
7			
8			
2			
6			
7			
8			
7			
9			
9			
L1(21) =			

Now press  $\boxed{\text{TAT}}$  move to the right to  $\boxed{\text{A}}$ , press  $\boxed{\text{ENTER}}$  to select 1-Var  $\boxed{\text{a}}$  (for one-variable statistics). Notice that the calculator returns to the home screen. Then press  $\boxed{\text{nd}}$   $\boxed{1}$  (above  $\boxed{1}$ )  $\boxed{\text{ENTER}}$ .

```
1-Var Stats L1
```

```
1-Var Stats
x̄=5.6
Σx=112
Σx²=710
Sx=2.087557122
σx=2.034698995
n=20
```

```
1-Var Stats
n=20
minX=2
Q1=4
Med=5.5
Q3=7
maxX=9
```

Use the down cursor to see all of the given information.

The sample mean is 5.6 and the sample standard deviation is about 2.09.

$\boxed{\text{nd}}$   $\boxed{\text{nd}}$  Find the mean and sample standard deviation for the given frequency data.

x	f
30	2
40	5
50	6
60	8
70	4
80	3
90	1

$\boxed{\text{nd}}$   $\boxed{\text{nd}}$   $\boxed{x}$   $\boxed{\text{nd}}$   $\boxed{\text{nd}}$   $\boxed{2}$  and the frequencies as  $\boxed{3}$ .

Check all values before continuing.

Now press  $\boxed{\text{TAT}}$  move to the right to  $\boxed{\text{A}}$ , press  $\boxed{\text{ENTER}}$  to select 1-Var  $\boxed{\text{a}}$  (for one-variable statistics).

Then press  $\boxed{\text{nd}}$   $\boxed{2}$  (above  $\boxed{2}$ )  $\boxed{\text{nd}}$   $\boxed{3}$  (above  $\boxed{3}$ )  $\boxed{\text{ENTER}}$ .

Use the down cursor to see all the given information.

```
L1  L2  L3  3
2   30  2
4   40  5
5   50  6
6   60  8
7   70  4
8   80  3
9   90  1
L3(1)=2
```

```
1-Var Stats L2,L
3
```

```
1-Var Stats
x̄=56.89655172
Σx=1650
Σx²=100500
Sx=15.37703489
σx=15.10958779
n=29
```

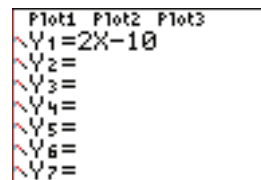
```
1-Var Stats
n=29
minX=30
Q1=45
Med=60
Q3=70
maxX=90
```

The sample mean is about 56.9 and the sample standard deviation is about 15.4.

### The Box-Whisker Plot

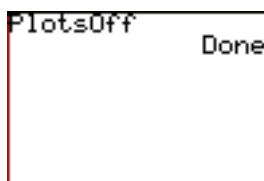
If you have data in a list, or frequency data in two lists, you can easily graph a box-whisker plot on the TI-84/83 Plus.

To prepare for a statistical plot, first be sure that you don't have any functions turned on. Either clear all of your functions in the  $y=$  window, or turn them off by moving the cursor on the top of the equal sign and pressing **ENTER**.



When the equal sign is no longer highlighted, the function will not be graphed.

It is also a good idea to turn off any previous statistical plots. You can do this by pressing **2<sup>nd</sup>** STAT PLOT (above  $y=$ ) **4** (for PlotsOff) **ENTER**. Or, in the  $y=$  window, you can move the cursor on top of any highlighted plots (at the top of the window) and press **ENTER** to turn them off.



To create a box-whisker plot, press **2<sup>nd</sup>** STAT PLOT (above  $y=$ ).

Select one of the three plots by typing the option number or moving the cursor to it and pressing **ENTER**. Then press **ENTER** to turn it on.

Then move to the box-whisker diagram under Type and press **ENTER**.

(The box-whisker diagram is in the middle of the second row. Move to Type and then move to the **right** to get to the box-whisker diagram.)

Type the name of the list for the Xlist.

If the data is **not** frequency data, type the number 1 for Freq.

If the data is **frequency data**, type the name of the frequency list for Freq.

Press **GRAPH** to see the box-whisker plot.


 Create a box-whisker plot for the data from Example 6.


Press  STAT PLOT (above ).



Press  to select .

Press  to turn it on.

Move to the box-whisker diagram and press .

Our data is in  $L_1$  and it is not frequency data, so the current information for list and Freq is fine as is.

Press  to see the box-whisker plot.

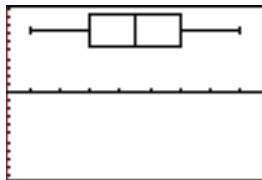
Press   (for ZoomStat – zoom statistics) to fit the window to the statistical data.

Press    to see the values for this new window.

```

ZOOM MEMORY
1:ZOut
2:ZDecimal
3:ZSquare
4:ZStandard
5:ZTri9
6:ZInteger
7:ZoomStat

```



```

WINDOW
Xmin=1.3
Xmax=9.7
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1


```



 Create a box-whisker plot for the frequency data from Example 7.

Press  STAT PLOT (above ).

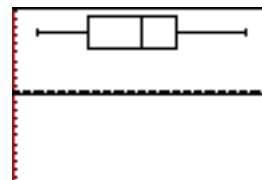
Press  to select Plot1.

Be sure that it is turned on and that the box-whisker diagram is selected.

Change the list to  $L_2$  and the Freq to  $L_3$ .

Press   (for ZoomStat – zoom statistics) to fit the window to the statistical data.

Press    to see the values for this new window.



```

WINDOW
Xmin=24
Xmax=96
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```

### The $\square$ STR Menu

The  $\square$ STR (distribution) menu gives values for the normal, student-t, Chi-square, F, binomial, Poisson and geometric probability distributions.

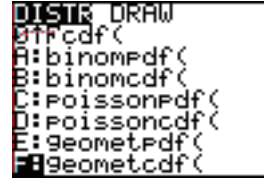
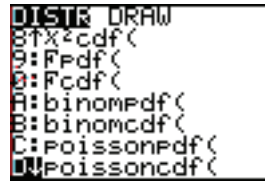
Here is the format to use with each one:

Distribution	Format
Normal probability density	$\text{nor}\square\square\text{pdf}(x\text{-value, mean } \mu, \text{ standard deviation } \sigma)$
Cumulative normal distribution probability	$\text{nor}\square\square\text{cdf}(\text{lower bound, upper bound, mean } \mu, \text{ standard deviation } \sigma)$
Inverse cumulative normal distribution	$\text{in}\square\text{Nor}\square(\text{area, mean } \mu, \text{ standard deviation } \sigma)$
Inverse T distribution*	$\text{in}\square\text{T}(\text{area, degrees of freedom})$
Student-t probability density	$\text{tpdf}(x\text{-value, degrees of freedom})$
Cumulative Student-t distribution	$\text{t}\square\text{cdf}(\text{lower bound, upper bound, degrees of freedom})$
Chi-square probability density	$\chi^2 \text{ pdf}(x\text{-value, degrees of freedom})$
Cumulative chi-square distribution function	$\chi^2 \square\text{cdf}(\text{lower bound, upper bound, degrees of freedom})$
F probability density	$\text{Fpdf}(x\text{-value, numerator degrees of freedom, denominator degrees of freedom})$
Cumulative F distribution function	$\text{F}\square\text{cdf}(\text{lower bound, upper bound, numerator degrees of freedom, denominator degrees of freedom})$
Binomial probability	$\square\text{ino}\square\text{pdf}(\text{number of trials, probability } p \text{ of success, } x\text{-value})$
Cumulative binomial probability	$\square\text{ino}\square\square\text{cdf}(\text{number of trials, probability } p \text{ of success, } x\text{-value})$
Poisson probability	$\text{poissonpdf}(\text{mean } \mu, x\text{-value})$
Cumulative Poisson probability	$\text{poisson}\square\text{cdf}(\text{mean } \mu, x\text{-value})$
Geometric probability	$\square\text{eo}\square\text{etpdf}(\text{probability } p \text{ of success, } x\text{-value})$
Cumulative geometric probability	$\square\text{eo}\square\text{et}\square\text{cdf}(\text{probability } p \text{ of success, } x\text{-value})$

\*This may or may not be available in your  $\square$ STR menu. It will depend upon the version of the operating system on your calculator.

To access the □STR menu, press **2<sup>nd</sup>** □STR (above □ARS ).

Use the up and down cursor arrows to see everything in the menu.



**10** Find the probability of getting exactly 8 heads in 10 tosses of a fair coin.

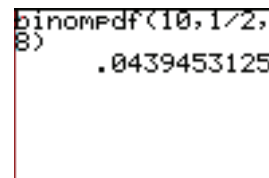
Press **2<sup>nd</sup>** **□**STR (above **□**ARS ) and scroll down to **ino□pdf□** (for binomial probability distribution) and press **ENTER** to select it.



Press   (for the number of times the coin is tossed)

☐ ☐ ☐ ☐ or ☐, for the probability of getting a head in one toss of a fair coin

☐ ENTER.



The probability of getting exactly 8 heads in 10 tosses of a fair coin is about 0.04.

**Example** Find the probabilities of getting 7, 8 or 9 heads in 10 tosses of a fair coin.

□his is a binomial probability.

Press  $\boxed{2^{nd}}$   $\boxed{\square}$ STR (above  $\boxed{\square}$ ARS ) and scroll down to  $\boxed{\square}$ ino $\boxed{\square}$ pdf $\boxed{\square}$  (for binomial probability distribution) and press  $\boxed{\square}$ ENTER $\boxed{\square}$  to select it.



Press  $\boxed{10} \boxed{1/2}$  (for the number of times the coin is tossed)

$\boxed{7} \boxed{8} \boxed{9} \boxed{\div} \boxed{2}$  (or  $\boxed{.5}$ , for the probability of getting a head in one toss of a fair coin)

$\boxed{2^{nd}}$  { (above  $\boxed{}$ )  $\boxed{}$   $\boxed{}$   $\boxed{}$   $\boxed{}$   $\boxed{}$   $\boxed{2^{nd}}$  } (above  $\boxed{}$ )  $\boxed{ENTER}$  .

$\boxed{\rightarrow}$  use the right cursor arrow to see the three probabilities.

```
binompdf(10,1/2,
(7,8,9))
.11171875 .0439...
```

```
binompdf(10,1/2,
(7,8,9))
.0439453125 ...
```

```
binompdf(10,1/2,
(7,8,9))
.125 .009765625)
```

The probability of getting exactly 7 heads in 10 tosses of a fair coin is about 0.12, the probability of getting exactly 8 heads in 10 tosses of a fair coin is about 0.04 and the probability of getting exactly 9 heads in 10 tosses of a fair coin is about 0.01.

You can find the sum of these probabilities by

pressing  $\boxed{2^{nd}}$  LIST (above  $\boxed{STAT}$ )  $\boxed{\rightarrow}$

moving to the right to select  $\boxed{ATH}$

$\boxed{}$  (for option  $\boxed{\rightarrow}$  sum( )

$\boxed{2^{nd}}$  ANS (above  $\boxed{(-)}$ , for the list of probabilities we found) and  $\boxed{ENTER}$  ,

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7:stdDev(
```

```
binompdf(10,1/2,
(7,8,9))
.125 .009765625)
sum(Ans)
.1708984375
```

The probability of getting 7, 8 or 9 heads in 10 tosses of a fair coin is about 0.17.

**Example 12** Find the probability of getting **at most** 4 heads in 10 tosses of a fair coin.

This is a cumulative binomial probability.

Press  $\boxed{2^{nd}}$  DISTR (above  $\boxed{VARS}$ ) and scroll

down to binomcdf( (for **cumulative** binomial probability distribution)

and press  $\boxed{ENTER}$  to select it.

```
DISTR DRAW
1:1-PropZTest(
2:1-PropZInt(
3:2-PropZTest(
4:2-PropZInt(
5:binompdf(
6:binomcdf(
7:Poissonpdf(
8:Poissoncdf(
```

Press  $\boxed{10} \boxed{0}$  (for the number of times the coin is tossed)

$\boxed{,} \boxed{1} \boxed{\div} \boxed{2}$  (or 0.5, for the probability of getting a head in one toss of a fair coin)

$\boxed{,} \boxed{4}$  (for getting at most 4 heads)  $\boxed{)}$   $\boxed{ENTER}$  .

```
binomcdf(10,1/2,
4)
.376953125
```

The probability of getting at most 4 heads in 10 tosses of a fair coin is about 0.38.



NOTE: You can find use the cumulative binomial probability distribution to find the probability of getting at least a certain number. For example, to find the probability of getting at least 5 heads in 10 tosses of a fair coin, we can subtract our answer from Example 12 from one. The probability is about 0.62.

```
binomcdf(10,1/2,
4)
.376953125
1-Ans
.623046875
```

☐☐☐☐ ☐☐☐ 1 Find the area from 85 to 110 under a normal distribution with mean  $\mu = 100$  and standard deviation  $\sigma = 15$ .

*This is a cumulative normal probability.*

Press **2<sup>nd</sup>** DISTR (above **VARS** ) and scroll  
down to **normal**( for **cumulative** normal probability distribution)  
and press **ENTER** to select it.

Press , , ,  (for the endpoints)

, 1 0 0 (for the mean  $\mu$ )

☐ , ☒ 1 ☐ (for the standard deviation  $\sigma$ )

) ENTER.

```
DISTR DRAW
1:normalPdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
```

```
normalcdf(85,110
,100,15)
.5888522734
```

The area (or probability) is about 0.59.

Find the 80<sup>th</sup> percentile for data that is normally distributed with  $\mu = 100$  and standard deviation  $\sigma = 15$ .

This is a normal distribution, but we want to use the percentile to find an  $x$ -value. So we need the inverse normal option.

Press **2<sup>nd</sup>** DISTR (above **VARS** ) and scroll  
down to **inNorm(** (for **i** **|||||** cumulative normal distribution)  
and press **ENTER** to select it.

Press    0 (for the 0<sup>th</sup> percentile)

$\boxed{\phantom{0}}, \boxed{1}, \boxed{0}, \boxed{0}$  (for the mean  $\mu$ )

☐ , ☒ 1 ☐ (for the standard deviation  $\sigma$ )

) ENTER.

```

DIST DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(

```

```
invNorm(.80,100,  
15)      112.6243185
```

The 80<sup>th</sup> percentile is about 112.6.

### The DISTR DRAW Menu

The DISTR DRAW (distribution draw) menu gives drawings of shaded regions on the distributions. Here is the format to use with each one:

Distribution	Command
Normal distribution	ShadeNorm(lower bound, upper bound, mean $\mu$ , standard deviation $\sigma$ )
Student-t distribution	ShadeT(lower bound, upper bound, degrees of freedom)
Chi-square distribution	Shade $\chi^2$ (area, mean $\mu$ , standard deviation $\sigma$ )
F distribution	ShadeF(area, degrees of freedom)

To access the DISTR DRAW menu,  
press  $2^{nd}$  DISTR (above  $\boxed{\text{VAR}}$ )  
and use the cursor to move to the right to DRAW.

```
DISTR DRAW
1:ShadeNorm(
2:ShadeT(
3:Shade $\chi^2$ (
4:ShadeF(
```

To clear the drawing,  
press  $2^{nd}$  DRAW (above  $\boxed{\text{RND}}$ )  
and  $\boxed{\text{ENTER}}$  (to select option 1 for  $\boxed{\text{DRAW}}$ )  
and  $\boxed{\text{ENTER}}$ .

```
DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Tangent(
6:DrawF
7:Shade(
```

**Example 1** Draw the region from 85 to 110 under a normal distribution with mean  $\mu = 100$  and standard deviation  $\sigma = 15$ .

Before drawing this, we need to set the window to show the distribution.

Press  $\boxed{\text{WINDOW}}$  to change the window settings.

With a mean of  $\mu = 100$ , we should center our window at 100.

Let's show three standard deviations to either side of the mean.

Because  $\sigma = 15$ , we would want to be  
 $3 * 15 = 45$  to either side of 100.

To set the values in the window  
from  $100 - 45 = 55$  to  $100 + 45 = 145$ .

A good choice for the scale on the x-axis is 0.

```
WINDOW
Xmin=55
Xmax=145
Xscl=20
Ymin=0
Ymax=.03
Yscl=.003
Xres=1
```

The total area under a normal distribution is only one, so y will be small.

Let's set the y-values from zero to 0.03 with a scale on the x-axis of 0.003.

Press  $\square\square$  and be sure that all functions and statistical plots are turned off.

See page  $\square\square$  if you need help with this.

Press  $2^{nd}$   $\square\square$ IT to return to the home screen

Now press  $2^{nd}$  DISTR (above  $\square$  VARS ) and use the cursor to move to the right to DRAW.

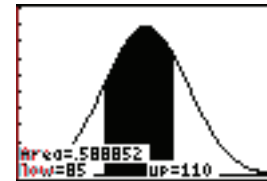
Press  $\square$ ENTER (to select option  $\square$  for ShadeNorm(.

$\square\square$  , 1 1 0 (for the endpoints)

1 0 0 (for the mean  $\mu$ )

, 1  $\square$  (for the standard deviation  $\sigma$ )

)  $\square$ ENTER.

Notice that the probability of about 0.59 is also given.

To clear the drawing, press  $2^{nd}$  DRAW (above  $\square$  R $\square\square$ ) and  $\square$ ENTER (to select option  $\square$  for  $\square$  D $\square\square$ ) and  $\square$ ENTER.

Remember to re-set your window values for another graph.

**Example 1** Draw the area under the standard normal distribution to the right of  $z = 1.27$

Remember that the standard normal distribution has a mean  $\mu = 0$  and standard deviation  $\sigma = 1$ . The TI-84 Plus uses this mean and standard deviation if no value is given for the mean and standard deviation.

Before drawing this, we need to set the window to show the distribution.

Press  $\square$ IND $\square\square$  to change the window settings.

With a mean of  $\mu = 0$ , we should center our window at 0.

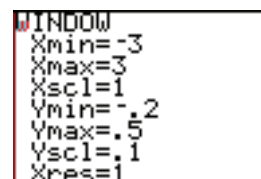
Let's show three standard deviations to either side of the mean.

Because  $\sigma = 1$ , we would want to be wide on either side of 0.

Let the  $x$ -values in the window from  $-3$  to  $3$ .

A good choice for the scale on the  $x$ -axis is  $\square$

Let's set the  $y$ -values from  $-0.02$  to  $0.5$  with a scale on the  $y$ -axis of  $0.1$ .



Press  $\square\square$  and be sure that all functions and statistical plots are turned off.  
 See page  $\square\square$  if you need help with this.

Press  $2^{nd}$   $\square\square$ IT to return to the home screen

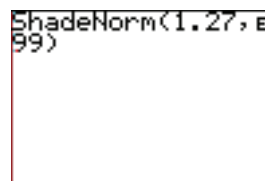
Now press  $2^{nd}$  DISTR (above  $\square$  VARS ) and use the cursor to move to the right to DRA $\square$ .

Press  $\square$ ENTER (to select option  $\square$  for  $\square\square\square$ d $\square$ Norm( .

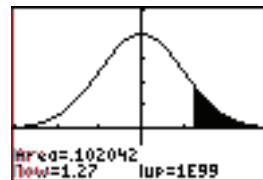
$\square$  1  $\square$  2  $\square$  (for the left endpoint)

$\square$  ,  $2^{nd}$  EE (above  $\square$  )  $\square$   $\square$  (for a very large number)

$\square$  )  $\square$ ENTER .



If you did not enter the mean and standard deviation.  
 When none are given, the TI-84 Plus uses the mean  $\mu = 0$  and the standard deviation  $\sigma = 1$  for the standard normal distribution.



Notice that the probability of about 0.10 is also given.

To clear the drawing, press  $2^{nd}$  DRA $\square$  (above  $\square$  R $\square\square$  )  
 and  $\square$ ENTER (to select option 1 for  $\square\square$ D $\square\square$ ) and  $\square$ ENTER .

Remember to re-set your window values for another graph.

### Confidence Intervals

To access the confidence interval options

press  $\square$ STAT and move to the right to TESTS .

Use the down cursor arrow to find the confidence intervals.



Select the one you want and press  $\square$ ENTER .

When the confidence interval window appears, select whether you are using data in a list or the statistics for the sample. Then type the other information needed.

When you are done entering information, move on top of  $\square\square$ C $\square\square\square$  (at the bottom of the screen) and press  $\square$ ENTER .

**Example 1** Find a 95% confidence interval for the population mean of a sample of size 40 with a sample mean of 55 and a sample standard deviation of 5.

Press **[STAT]** and move to the right to TESTS .

Move down to ZInterval... and press **[ENTER]** to select it.

We are given statistics (the sample size, mean and standard deviation), so we want to select the Stats (statistics) Inpt (input).

Move to the right to Stats and press **[ENTER]** to select it.

Type the values for  $\sigma$  (the sample standard deviation),  $\bar{x}$  (the sample mean) and  $n$  (the sample size).

Then type the value for the C-Level (confidence level).

Move down to Calculate and press **[ENTER]** to calculate the confidence interval.

A 95% confidence interval for the mean is (53.41, 56.549).

```

EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...

```

```

ZInterval
Inpt: Stats
σ: 5
List: L1
Freq: 1
C-Level: .95
Calculate

```

```

ZInterval
Inpt: Data Stats
σ: 5
x: 55
n: 40
C-Level: .95
Calculate

```

```

ZInterval
(53.451, 56.549)
x=55
n=40

```

**Example 18** Find a 90% confidence interval for this sample of body temperatures {98.6, 98.0, 99.0, 98.4, 98.6, 98.0, 98.4}.

First enter the data in a list. Let's use  $L_1$ .

See "Entering Data in a List" if you need help.

L1	L2	L3	1
98.6			
98			
99			
98.4			
98.6			
98			
98.4			

L1(1)=98.6

This is a small sample, so we need to use a t-interval rather than a z-interval.

Press **[STAT]** and move to the right to TESTS .

Move down to TInterval... and press **[ENTER]** to select it.

```

EDIT CALC TESTS
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
8:TInterval...
9:2-SampZInt...

```

Move to  $\square \Delta \square$  and

press  $\square \text{ENTER}$  to select it.

Be sure that the List is  $L_1$ .

The data is not frequency data, so Freq should be 1.

Type the value for the C-Level (confidence level).

Move down to Calculate and press  $\square \text{ENTER}$   
to calculate the confidence interval.

A 90 % confidence interval for the mean is (98.168, 98.689).

Notice that the sample mean, sample standard deviation and sample size are also given.

```
Interval
Inpt: Data Stats
List: L1
Freq: 1
C-Level: .9
Calculate
```

```
Interval
(98.168, 98.689)
x=98.42857143
Sx=.3545621042
n=7
```

### Hypothesis Tests

To access the hypothesis test options

press  $\square \text{STAT}$  and move to the right to TESTS .

Use the down cursor arrow to find the hypothesis tests.

```
EDIT CALC TESTS
1: Z-Test...
2: T-Test...
3: 2-SampZTest...
4: 2-SampTTest...
5: 1-PropZTest...
6: 2-PropZTest...
7: ZInterval...
```

Select the one you want and press  $\square \text{ENTER}$  .

When the hypothesis test window appears, select whether you are using data in a list or the statistics for the sample. Then type the other information needed.

When you are done entering information, move on top of Calculate (at the bottom of the screen) and press  $\square \text{ENTER}$  .

$\square \square \square \square \square \square 1 \square$  Test the following hypothesis:  $\square_0 : \mu_0 = 4$

$\square_1 : \mu > 4$

given a sample mean of 5.5, a sample standard deviation of 2  
and a sample size of 40.

Press **[STAT]** and move to the right to TESTS .

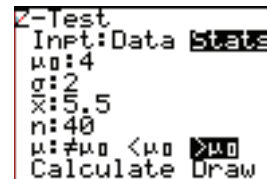
Move down to Z-Test... and press **[ENTER]** to select it.



Move to the right to Stats and

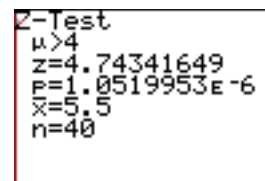
press **[ENTER]** to select it.

Type the values for  $\mu_0$ ,  $\sigma$  (the sample standard deviation),  $\bar{x}$  (the sample mean) and  $n$  (the sample size).



Then move down and across to  $> \mu_0$  to test the hypothesis that the true mean is larger than  $\mu_0$  and press **[ENTER]** to select it.

Move down to Calculate and press **[ENTER]** to calculate the confidence interval.



Notice that we are given the  $p$ -value as well as the  $z$ -value.

### Statistics – Two Variable Data

The TI-84/83 Plus can quickly calculate the following for a given list of two-variable (paired) values:

<b>[2nd] [e] [2nd] [a] [     ]</b>	<b><math>x</math></b>	<b><math>y</math></b>
The mean	$\bar{x}$	$\bar{y}$
The sum of the values	$\sum x$	$\sum y$
The sum of the squares of the values	$\sum x^2$	$\sum y^2$
The sample standard deviation	$S_x$	$S_y$
The population standard deviation	$\sigma_x$	$\sigma_y$
The minimum of the values	<b>[2nd] [n]</b>	<b>[2nd] [n]</b>
The maximum of the values	<b>[2nd] [a] X</b>	<b>[2nd] [a] Y</b>
The number of data points	$n$	
The sum of the products of the values	$\sum xy$	

To get this information about a list, first type your data in two lists.

Then press **STAT**, move to the right to CALC, press **□** to select

**2-Var Statistics** (for two-variable statistics), then type the list names, separated by a comma, and press **ENTER**.

**Example** **□□** The data in the table gives the number of hours 13 students studied paired with the score each student received on an exam. Find the statistics for this data.

<b>□ □ □ □ □ □ □ □ □ □ □ □ □ □</b>	<b>Exam □ □ □ □ □</b>
5	98
4	94
5	92
4	85
3	80
4	78
3	75
2	74
3	74
2	72
1	68
2	62
1	58

Type the data using  $L_1$  for the hours studied and  $L_2$  for the exam scores.

Check your lists carefully for accuracy before continuing.

<b>L1</b>	<b>L2</b>	<b>L3</b>	<b>2</b>
5	98	---	---
4	94	---	---
5	92	---	---
4	85	---	---
3	80	---	---
4	78	---	---
3	75	---	---
<b>L2(1)=98</b>			

<b>L1</b>	<b>L2</b>	<b>L3</b>	<b>2</b>
2	74	---	---
2	74	---	---
2	72	---	---
2	68	---	---
2	62	---	---
1	58	---	---
<b>L2(14)=</b>			

Press **□** **QUIT** to return to the home screen

Press **STAT** move to the right to CALC,

press **□** to select

**2-Var Statistics** (for two-variable statistics),

then type the list names, separated by a

comma, and press **ENTER**.

<b>2-Var Stats L1,L</b>
<b>2</b>



Use the down cursor to see the information given.

```
2-Var Stats
x=3
Σx=39
Σx²=139
Sx=1.354006401
σx=1.300887271
n=13
```

```
2-Var Stats
y=77.69230769
Σy=1010
Σy²=80206
Sy=12.03041019
σy=11.55844424
Σxy=3207
```

```
2-Var Stats
r=11.55844424
Σxy=3207
minX=1
maxX=5
minY=58
maxY=98
```

### The Scatter Plot

If you have two-variable data in two lists you can easily graph a scatter plot on the TI-84/83 Plus.

To prepare for a statistical plot, first be sure that you don't have any functions turned on. Either clear all of your functions in the  $y=$  window, or turn them off by moving the cursor on the top of the equal sign and pressing **ENTER**.

```
Plot1 Plot2 Plot3
Y1=2X-10
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

When the equal sign is no longer highlighted, the function will not be graphed.

It is also a good idea to turn off any previous statistical plots. You can do this by pressing **2ND** **STAT** **2** **LO** **T** (above **2**) **2** (for **Plots**) **ENTER**. Or, in the  $y=$  window, you can move the cursor on top of any highlighted plots (at the top of the window) and press **ENTER** to turn them off.

```
STAT PLOTS
1:Plot1...Off
   L1 L2
2:Plot2...Off
   L1 L2
3:Plot3...Off
   L1 L2
4:PlotsOff
```

```
PlotsOff Done
```

To create a scatter plot, press **2ND** **STAT** **2** **LO** **T** (above **2**) **2**.

Select one of the three plots by typing the option number or moving the cursor to it and pressing **ENTER**. Then press **ENTER** to turn it on.

Then move to the scatter diagram (the first option) under **Type** and press **ENTER**.

Type the names of the lists for the **Xlist** (the independent variable) and the **Ylist** (the dependent variable.)

Select one of the three marks at the bottom of the screen.

Then press **ZOOM** **9** (ZoomStat – zoom statistics) to see the scatter plot in a window using the given data.

**Example 21** Create a scatter plot for the data in Example 20.

Press  $\boxed{\text{2ND}} \boxed{\text{MODE}}$  (above  $\boxed{\text{MODE}}$ ).

Press  $\boxed{\text{F1}}$  to select  $\boxed{\text{Plot}}$ .

Press  $\boxed{\text{F1}}$  to turn it on.

Move to the scatter diagram and press  $\boxed{\text{F1}}$ .

Be sure that the Xlist is  $L_1$  and the Ylist is  $L_2$ .



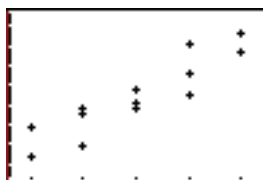
Press  $\boxed{\text{ZOOM}} \boxed{9}$  (for ZoomStat – zoom statistics) to fit the window to the statistical data.

Press  $\boxed{\text{2ND}} \boxed{\text{MEMO}}$  to see the values for this window.

```

2001 MEMORY
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9:ZoomStat

```



```

WINDOW
Xmin=.6
Xmax=5.4
Xscl=1
Ymin=51.2
Ymax=104.8
Yscl=1
Xres=1

```

### Regression Models

The TI-84/83 Plus can find regression models for the following **polynomials**:

1 <sup>st</sup> degree	$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F1}}$	linear regression	$y = ax + b$
1 <sup>st</sup> degree	$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F2}}$	linear regression	$y = a + bx$
2 <sup>nd</sup> degree	$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F3}}$	quadratic regression	$y = ax^2 + bx + c$
3 <sup>rd</sup> degree	$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F4}}$	cubic regression	$y = ax^3 + bx^2 + cx + d$
4 <sup>th</sup> degree	$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F5}}$	quartic regression	$y = ax^4 + bx^3 + cx^2 + dx + e$

It can also find these other regression models:

$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F6}}$	logarithmic regression	$y = a + b \ln x$
$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F7}}$	exponential regression	$y = ab^x$
$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F8}}$	power regression	$y = ax^b$
$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F9}}$	logistic regression	$y = \frac{c}{1 + ae^{-bx}}$
$\boxed{\text{2ND}} \boxed{\text{MODE}} \boxed{\text{F1}} \boxed{\text{F0}}$	sinusoidal regression	$y = a \sin(bx + c) + d$

To create a regression model,  
press **S□□□** move to **□□□□** ,  
and select one of the types of regression.  
Type the names of the lists separated  
by a comma and press **□□□□□** .

```

EDIT CALC TESTS
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg

```

```













EDIT  CALC  TESTS
7↑QuartReg
8:LinReg(a+bx)
9:LnReg
10:ExpReg
H:PwrReg
B:Logistic
C↓SinReg

```

If you would like to see the linear correlation coefficient  $r$  and the coefficient of determination  $r^2$  for some models, you need to turn the diagnostics on.

```
CATALOG
Degree
DelVar
DependAsk
DependAuto
det(
DiagnosticOff
DiagnosticOn
```

To do this, press

         (above  , the zero key)  
 (notice that the cursor is in ALPHA mode)  
 and  (above   $x^{-1}$ )

DiagnosticOn Done

move down to **ia** **osti** **O** and  
press **□□□□□** to select it and then  
press **□□□□□** again to turn the diagnostics on.

**Example 22** Find the equation and the graph for the data in Example 20. Predict an exam score for a student who studies for 2.5 hours.

*Be sure that you have turned the diagnostics on (see above.)*

Press **S**  $\rightarrow$  mo  $\rightarrow$  e to  $\rightarrow$  ,  
 press  $\rightarrow$  (for linear regression  $\rightarrow$  i  $\rightarrow$   $\rightarrow$  a  $\rightarrow$  )  
 and press  $\rightarrow$  .

EDIT CALC TESTS  
1:1-Var Stats  
2:2-Var Stats  
3:Med-Med  
4:LinReg(ax+b)  
5:QuadReg  
6:CubicReg  
7:QuartReg

Type the names of the lists separated by a comma and press

```

EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg

```

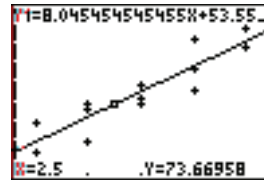
LinReg(ax+b) L1,  
L2

```
LinReg
y=ax+b
a=8.045454545
b=53.55594406
r^2=.8199393617
r=.9055050313
```

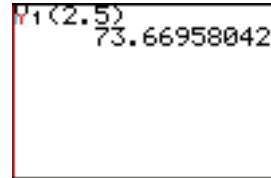
This gives us the regression model of approximately  $y = 8.045x + 53.556$ .



to predict an exam score for a student who studied  $x$  hours either press  $\square\square\square\square\square$  and type the value  $x$  and press  $\square\square\square\square\square$



or return to the home screen and press  $\square\square\square\square\square$  (for variables) move to the right to  $\square\square\square\square\square$  press  $\square\square\square\square\square$  (to select  $y_1$ ) and press  $\square\square\square\square\square$  (to select  $y_1$ ) type the value  $x$  and press  $\square\square\square\square\square$ .



## 10 Calculus

### Finding Local (Relative) Extrema over an Interval Using the MATH Key

One way to find the local (relative) maximum or minimum value of a function over a closed interval is to use the **MATH** key.

Type the function in the y= window first.

Then press **MATH** and

select **fMin(** (for function local minimum)

or **fMax(** (for function local maximum) to find

the  $x$ -value where the local or relative minimum or maximum occurs.

Then evaluate the function at that  $x$ -value to find the local minimum or local maximum function value.



The format to use is:

**fMin**(the expression, the variable, the smallest value in the interval, the largest value in the interval)

**fMax**(the expression, the variable, the smallest value in the interval, the largest value in the interval)

**Example 1** Find the local minimum and local maximum values for

$$f(x) = \frac{x^3 - 12x}{8} \text{ in the closed interval } [-3, 5].$$

First type the function in the y= window.  
(Be sure that all statistical plots are off.)

Press **2<sup>nd</sup>** **QUIT** to return to the home screen.

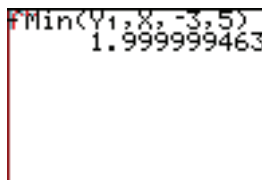
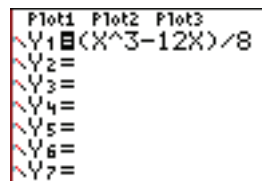
Now press **MATH** **6** (to select **fMin(** ).

Press **VAR** (for variables)

move to the right to **Y-VARS**

press **ENTER** (to select **Function...**)

and press **ENTER** (to select  $y_1$ ).



Press **,** **x** **,** **(-)** **3** **,** **5** **)** **ENTER**.

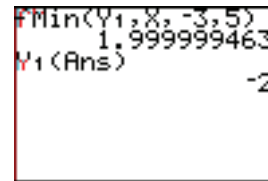
This tells us that the local minimum  $y$ -value occurs when  $x$  is about 2.

□o find the local minimum  $y$ -value on this interval□

press **[VARS]** move to the right to Y-VARS□

press **[ENTER]** (to select Function...)

and press **[ENTER]** (to select  $y_1$ ).



□hen press **[ ]** **[2<sup>nd</sup>]** **[ANS]** (to use the answer we found for the  $x$ -value)

and **[ ) ]** **[ENTER]**.

The local minimum  $y$ -value in this interval is  $-2$ .

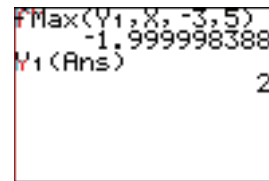
□ou can repeat this procedure using  $fMax()$  to find the maximum  $y$ -value.

To speed up the process, press **[2<sup>nd</sup>]** **[ENTRY]** *twice*

to return the  $fMin()$  line to the screen.

Move on top of  $fMin()$  and replace it

with  $fMax()$  and then press **[ENTER]**.



Then press **[2<sup>nd</sup>]** **[ENTRY]** *twice*

and **[ENTER]** to evaluate the function at that  $x$ -value

The local maximum  $y$ -value in this interval is  $2$ .

**WARNING:** These options do **not** find **the extrema** over the interval. The smallest  $y$ -value in this interval is  $2$ , but the largest  $y$ -value occurs at the endpoint  $x = 5$ . This maximum value is  $y = 8.125$ .

### Finding a Numerical Derivative Using the MATH Key

One way to find a numerical derivative of a function is to use the **[MATH]** key and the  $nDeriv()$  option (numerical derivative).

Use the format

$nDeriv()$  ( the function, the variable, the value)



Find the derivative of  $r^2$  when  $a = \pi$ .

**MATH** **8** to select nDeriv( for numerical derivative.

Now press **ALPHA** **R**  $x^2$  ,

**ALPHA** **R** ,

**2<sup>nd</sup>**  **$\pi$**  **)** **ENTER**.

```
nDeriv(R2,R, $\pi$ )
6.283185307
```

The derivative is approximately 6.28.

Find  $\frac{dy}{dx}$  if  $y = \frac{x^3 - 12x}{8}$  and  $x = 3$ .

press **MATH** **8** to select nDeriv(

We have already stored this function as  $y_1$ , so

press **VAR** for variables

move to the right to Y-VARS,

press **ENTER** to select Function...

and press **ENTER** to select  $y_1$ .

```
nDeriv(Y1,X,3)
1.875000125
```

press **,** **X** **,** **3** **)** **ENTER**.

The derivative is approximately 1.875.

**WARNING:** The TI-84/83 Plus uses the symmetric difference quotient method – an approximation method – to find this numerical derivative. Be aware that this is not an exact method. Also be aware that it can return a false derivative value at a non-differentiable point.



### Finding a Definite Integral Using the MATH Key

One way to find a definite integral of a function is to use the **MATH** key and the **fnInt(** option (function integral).



Use the format **fnInt(** the function, the variable, the lower limit of integration, the upper limit of integration)

Find  $\int_0^2 \frac{x^3 - 12x}{8} dx$ .

press **MATH** **9** to select **fnInt(**

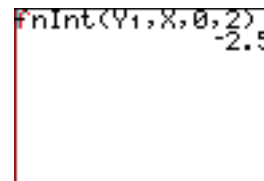
We have already stored this function as  $y_1$ , so

press **VAR** for variables

move to the right to Y-VARS,

press **ENTER** to select **Function...**

and press **ENTER** to select  $y_1$



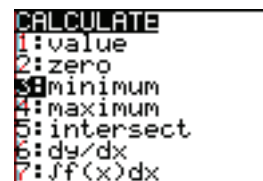
press **(, x , 0 , 2 ) ENTER**.

The integral  $\int_0^2 \frac{x^3 - 12x}{8} dx$  has an approximate value of  $-2.5$ .

**WARNING:** The TI-84/83 Plus uses the Gauss-Kronrod method – an approximation method – to find this numerical integral. Be aware that this is not an exact method.

### Finding Extrema over an Interval Using **2<sup>nd</sup>** CALC

If you have graphed your function, you can use **2<sup>nd</sup>** CALC (for calculate) to find a function's minimum or maximum value over an interval.



First graph your function in a window that shows the interval.  
Look for the local (relative) extrema within the interval.

Then press  $\boxed{2^{\text{nd}}}$  CALC and select either minimum or maximum.

Follow the directions on the screen, choosing a left bound, a right bound and a guess and then pressing  $\boxed{\text{ENTER}}$ .

You can select the values by typing the number and pressing  $\boxed{\text{ENTER}}$  or by moving the cursor to the desired location and pressing  $\boxed{\text{ENTER}}$ .

$\square\square\square\square\square\square\square$  Find the local minimum and local maximum values for

$$f(x) = \frac{x^3 - 12x}{8} \text{ in the closed interval } [-3, 5].$$

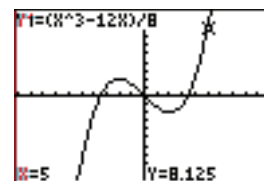
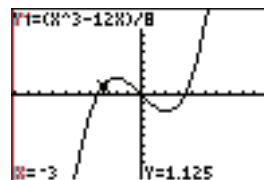
$\square$  e have already stored this function as  $y_1$ .  $\square\square$  ee  $\square$  xa  $\square$  ple  $\square\square$   
 $\square$  raph it in a standard window.

$\square$  ress  $\boxed{\text{TRACE}}$   $\boxed{(-)}$   $\boxed{3}$   $\boxed{\text{ENTER}}$

to find the value at the left endpoint of the interval.

$\square$  ress  $\boxed{5}$   $\boxed{\text{ENTER}}$

to find the value at the right endpoint of the interval.

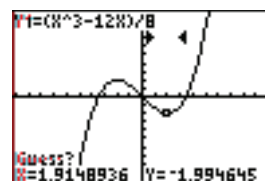
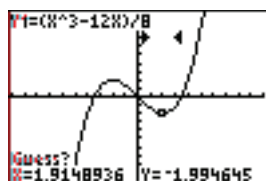
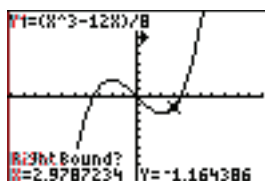
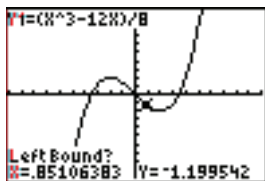


To find the local  $\square$  relative  $\square\square$  ini  $\square$  u  $\square$ , press  $\boxed{2^{\text{nd}}}$  CALC  $\boxed{3}$  for  $\square$  ini  $\square$  u  $\square$ ,

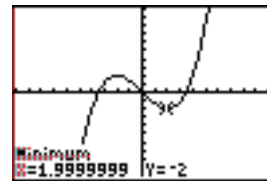
$\square$  ove to the left of the local  $\square$  ini  $\square$  u  $\square$  and press  $\boxed{\text{ENTER}}$ ,

$\square$  ove to the ri  $\square$  ht of the local  $\square$  ini  $\square$  u  $\square$  and press  $\boxed{\text{ENTER}}$ ,

and  $\square$  ove as close as possi  $\square$  le to the local  $\square$  ini  $\square$  u  $\square$  and press  $\boxed{\text{ENTER}}$  a  $\square$  ain.

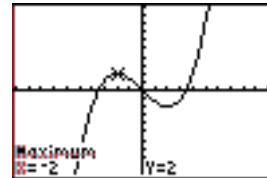


The screen now shows the cursor location of the local minimum, and the coordinates of the point.



Repeat this process to find the local maximum.

Now compare the endpoint y-values with the values at the local extrema.

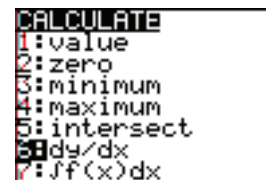


The maximum value for this function on the interval  $[-3, 5]$  is 8.125 at  $x = 5$  and the minimum value is  $-2$  at  $x = 2$ .

Compare this to our results in Example 1.

### Finding a Numerical Derivative Using $2^{nd}$ CALC

If you have graphed your function, you can use  $2^{nd}$  CALC (for calculate) to find a function's numerical derivative at a given x-value.



First graph your function in a window that shows the x-value.

Then press  $2^{nd}$  CALC and select dy/dx.

Select an x-value by moving to it or typing a number.

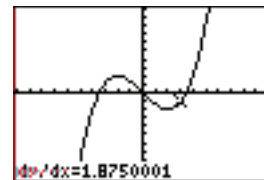
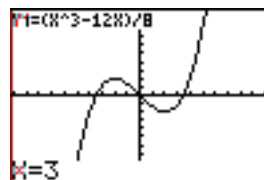
Then press **ENTER**.

Find  $\frac{dy}{dx}$  if  $y = \frac{x^3 - 12x}{8}$  and  $x = 3$ .

We have already stored this function as  $y_1$  and graphed it in a standard window.

Press  $2^{nd}$  CALC **6** for dy/dx, the numerical derivative **3** **ENTER**,

The derivative is approximately 1.875.

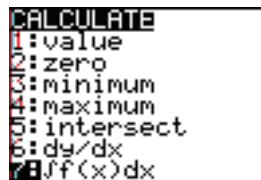


### Finding a Definite Integral Using $2^{\text{nd}}$ CALC

If you have graphed your function, you can use

$2^{\text{nd}}$  CALC (for calculate) to find a definite integral of a function.

First graph your function in a window that shows the lower and upper limits of integration.



The press  $2^{\text{nd}}$  CALC and select  $\int f(x)dx$ .

Select a lower limit by moving to it or typing a number and pressing  $\text{ENTER}$ .

Select an upper limit by moving to it or typing a number and pressing  $\text{ENTER}$ .

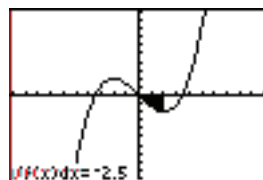
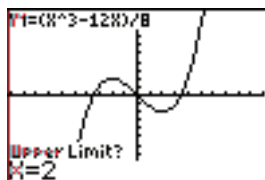
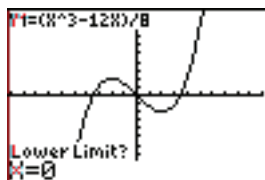
The calculator shades the region and gives the value of the definite integral.

$\square\square\square\square\square\square\square$  Find  $\int_0^2 \frac{x^3 - 12x}{8} dx$ .

$\square$  e have already stored this function as  $y_1$  and  $\square$ raphed it in a standard window.

To find the nu $\square$ eric inte $\square$ ral, press  $2^{\text{nd}}$  CALC  $7$  for definite inte $\square$ ral.

$0$   $\text{ENTER}$  for the lower li $\square$ it,  $2$   $\text{ENTER}$  for the upper li $\square$ it.



The integral  $\int_0^2 \frac{x^3 - 12x}{8} dx$  has an approximate value of  $-2.5$ .

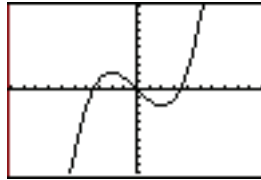
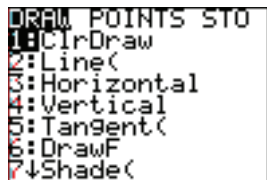
$\square$ o $\square$ pare this to our result in  $\square$ xa $\square$ ple  $\square$

To clear the drawin $\square$

press  $2^{\text{nd}}$  DRAW  $\square$ a $\square$ ove  $\text{PRGM}$   $\square$

and  $\text{ENTER}$  to select option  $\square$

for ClrDraw $\square$ and  $\text{ENTER}$ .





Graph  $y = f'(x)$  if  $f(x) = \frac{x^3 - 12x}{8}$ .

First type the function in the  $y=$  window.  
Move the cursor to  $y_2$ .

Then press **MATH** **8** to select  $nDeriv($ .

press **VARS** for variables move to the right to Y-VARS,  
press **ENTER** to select Function... and press **ENTER** to select  $y_1$ .

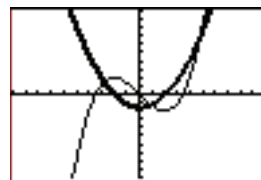
press **,** **X** **,** **X** **)**.

Move to the left of the  $y_2$  and  
press **ENTER** to create a heavier  
line for the graph of the derivative.

Now press **ZOOM** **9**

to graph the function and its  
numeric derivative in a standard window.

Plot1 Plot2 Plot3  
Y1=(X^3-12X)/8  
Y2=nDeriv(Y1,X)  
Y3=  
Y4=  
Y5=  
Y6=



### Derivatives – Polar Functions

If you have graphed a polar function,  
you can use the TI-84/83 Plus to find the  
numeric derivatives  $\frac{dy}{dx}$  and  $\frac{dr}{d\theta}$  using **2<sup>nd</sup>** **CALC**.

CALCULATE  
1:value  
2:dy/dx  
3:dr/dtheta

Find the approximate value of  $\frac{dr}{d\theta}$  at  $\theta = \frac{\pi}{3}$  if

$$r = 6 \cos^4 \theta + 4 \sin 3\theta.$$

Be sure that your calculator is in polar mode and radian mode.  
Graph the function in a standard window. (See Section 4, Example 7.)