Big Ideas Math: Geometry

Chapter Summary

Chapter 1: Basics of Geometry

Standards

Common Core:
HSG-CO.A.1, HSG-CO.D.12, HSG-MG.A.1, HSG-GPE.B.7

Learning Goals

Name points, lines, and planes.

Name segments and rays.

Sketch intersections of lines and planes.

Solve real-life problems involving lines and planes.

Use the Ruler Postulate.

Copy segments and compare segments for congruence.

Use the Segment Addition Postulate.

Find segment lengths using midpoints and segment bisectors.

Use the Midpoint Formula.

Use the Distance Formula.

Classify polygons.

Find perimeters and areas of polygons in the coordinate plane.

Name angles.

Measure and classify angles.

Identify congruent angles.

Use the Angle Addition Postulate to find angle measures.

Bisect angles.

Identify complementary and supplementary angles.

Identify linear pairs and vertical angles.

Core Vocabulary

Words that do not have formal definitions, but there is agreement about what they mean are called ***undefined term.***

A ***point*** is a location in space that is represented by a dot and has no dimension.

A ***line*** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

A flat surface made up of points that has two dimensions and extends without end and is represented by a slanted four-sided figure is a ***plane.***

Collinear points are points that lie on the same line.

Coplanar points are points that lie in the same plane.

Terms that can be described using known words are called ***defined terms.***

A ***line segment***, or ***segment*** consists of two endpoints and all the points between them.

Points that represent the ends of a line segment or ray are called ***endpoints.***

$\overrightarrow{AB}$ is a ***ray*** if it consists of the endpoint A and all points on $\overrightarrow{AB}$ that lie on the same side of A as B.

If point C lies on $\overleftrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are ***opposite rays***.

The set of points two or more geometric figures have in common is their ***intersection.***

A ***postulate*** is a rule that is accepted without proof.

An ***axiom*** is a rule that is accepted without proof.

A real number that corresponds to a point on a line is a ***coordinate.***

Distance is the absolute value of the difference of two coordinates on a line.

A ***construction*** is a geometric drawing that uses a limited set of tools, usually a compass and a straightedge.

Line segments that have the same length are ***congruent segments.***

When three points are collinear, one point is ***between*** the other two.

The point that divides a segment into two congruent segments is the ***midpoint.***

A point, ray, line, line segment, or plane that intersects the segment at its midpoint is a ***segment bisector.***

An ***angle*** is a set of points consisting of two different rays that have the same endpoint

The common endpoint of two rays is the ***vertex*** of an angle.

The ***sides of an angle*** are the rays of the angle.

The region that contains all the points between the sides of an angle is the ***interior of an angle.***

The region that contains all the points outside of an angle is the ***exterior of an angle.***

The absolute value of the difference between the real numbers matched with the two rays that form the angle on a protractor is the ***measure of an angle.***

An angle that has a measure greater than 0° and less than 90° is an ***acute angle.***

An angle that has a measure of 90° is a ***right angle.***

An angle that has a measure greater than 90° and less than 180° is an ***obtuse angle.***

An angle that has a measure of 180° is a ***straight angle.***

Two angles that have the same measure are ***congruent angles.***

A ray that divides an angle into two angles that are congruent is an ***angle bisector.***

Two angles whose measures have a sum of 90° are ***complementary angles.***

Two angles whose measures have a sum of 180° are ***supplementary angles.***

Adjacent angles are two angles that share a common vertex and side, but have no common interior points.

Two adjacent angles whose noncommon sides are opposite rays are a ***linear pair.***

Vertical angles are two angles whose sides form two pairs of opposite rays.

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Essential Questions

How can you use dynamic geometry software to visualize geometric concepts?

How can you measure and construct a line segment?

How can you find the midpoint and length of a line segment in a coordinate plane?

How can you find the perimeter and area of a polygon in a coordinate plane?

How can you measure and classify an angle?

How can you describe angle pair relationships and use these descriptions to find angle measures?

Postulates

1.1 Ruler Postulate
The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point. The distance between points *A* and *B*, written as *AB*, is the absolute value of the difference of the coordinates of *A* and *B*.

1.2 Segment Addition Postulate
If *B* is between *A* and *C*, then *AB* + *BC* = *AC*.
If *AB* + *BC* = *AC*, then *B* is between *A* and *C*.

1.3 Protractor Postulate
Consider $\overleftrightarrow{OB}$ and a point *A* on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180. The measure of ∠*AOB*, which can be written as *m*∠*AOB*, is equal to the absolute value of the difference between the real numbers matched with $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on a protractor.

1.4 Angle Addition Postulate
If *P* is in the interior of ∠*RST*, then the measure of ∠*RST* is equal to the sum of the measures of ∠*RSP* and ∠*PST*.

Core Concept

Undefined Terms: Point, Line, and Plane

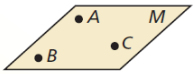
Point A point has no dimension. A dot represents a point.

point *A*

Line A line has one dimension. It is represented by a line with two arrowheads, but it extends without end. Through any two points, there is exactly one line. You can use any two points on a line to name it.

line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

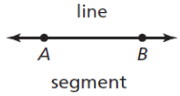
Plane A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



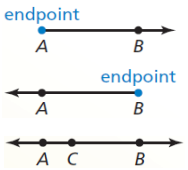
plane *M*, or plane *ABC*

Defined Terms: Segment and Ray

The definitions below use line *AB* (written as $\overleftrightarrow{AB}$) and points *A* and *B*.



Segment The line segment *AB*, or segment *AB*, (written as $\overline{AB}$) consists of the endpoints *A* and *B* and all points on $\overleftrightarrow{AB}$ that are between *A* and *B*. Note that $\overline{AB}$ can also be named $\overline{BA}$.



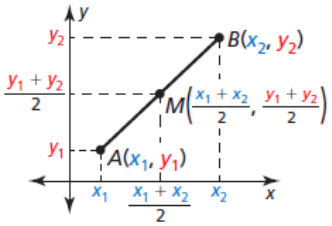
Ray The ray *AB* (written as $\overrightarrow{AB}$) consists of the endpoint *A* and all points on $\overleftrightarrow{AB}$ that lie on the same side of *A* as *B*. Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.

Opposite Rays If point *C* lies on $\overleftrightarrow{AB}$ between *A* and *B*, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the *x*-coordinates and of the *y*-coordinates of the endpoints. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint *M* of $\overline{AB}$ has coordinates

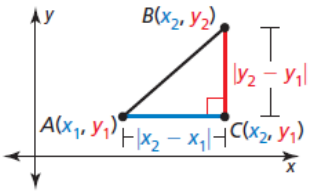
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

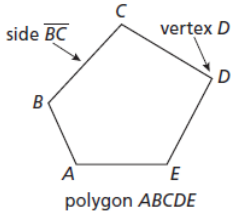


The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between *A* and *B* is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$





polygon *ABCDE*

Polygons

In geometry, a figure that lies in a plane is called a plane figure. Recall that a *polygon* is a closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two sides, one at each *vertex*, so that no two sides with a common vertex are collinear. You can name a polygon by listing the vertices in consecutive order.

Congruent Segments

- Line segments that have the same length are called congruent segments.
- You can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$."
- The symbol ≅ means "is congruent to."

Midpoints and Segment Bisectors

- The midpoint of a segment is the point that divides the segment into two congruent segments.
- A segment bisector is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.
- A midpoint or a segment bisector bisects a segment.

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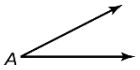
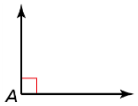
Core Concept

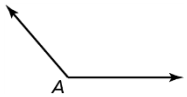
Types of Angles



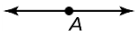
Acute Angle
Measures greater than 0° and less than 90°



Right Angle
Measures 90°



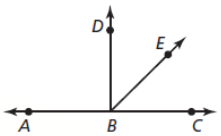
Obtuse Angle
Measures greater than 90° and less than 180°



Straight Angle
Measures 180°

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you ***can* conclude** from the diagram below.



YOU CAN CONCLUDE

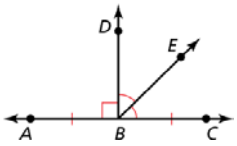
- All points shown are coplanar.
- Points *A*, *B*, and *C* are collinear, and *B* is between *A* and *C*.
- $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{BE}$ intersect at point *B*.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the interior of $\angle DBC$.

Here are some things you ***cannot* conclude** from the diagram above.

YOU CANNOT CONCLUDE

- $\overline{AB} \cong \overline{BC}$.
- $\angle DBE \cong \angle EBC$.
- $\angle ABD$ is a right angle.

To make such conclusions, the following information must be given.



Linear Pairs and Vertical Angles

- Two adjacent angles are a linear pair when their noncommon sides are opposite rays.
- The angles in a linear pair are supplementary angles.
- Two angles are vertical angles when their sides form two pairs of opposite rays.

Complementary and Supplementary Angles

Complementary Angles

- Two positive angles whose measures have a sum of 90°. Each angle is the *complement* of the other.

Supplementary Angles

- Two positive angles whose measures have a sum of 180°. Each angle is the *supplement* of the other.

Adjacent Angles

- Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*.
- Adjacent angles are two angles that share a common vertex and side, but have no common interior points.

Games

- What's the Angle?
- Race for Distance

These are available online in the *Game Closet* at www.bigideasmath.com.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 1: Alamillo Bridge STEM Video is available online at www.bigideasmath.com.

Additional Review

- Intersections of Lines and Planes, *p. 6*
- Classifying Polygons, *p. 30*
- Finding Perimeter and Area in the Coordinate Plane, *p. 31*
- Bisecting Angles, *p. 42*

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