Big Ideas Math: Algebra 2

Chapter Summary

Chapter 3: Quadratic Equations and Complex Numbers

Core Vocabulary

A ***quadratic equation in one variable*** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

A ***root of an equation*** is a solution of the equation.

A ***zero of a function*** f is an x-value for which $f(x) = 0$.

The ***imaginary unit i*** is the square root of -1, denoted $i = \sqrt{-1}$.

A ***complex number*** is a number written in the form $a + bi$, where a and b are real numbers.

A number written in the form $a + bi$, where a and b are real numbers and $b \neq 0$ is an ***imaginary number***.

A number written in the form $a + bi$, where $a = 0$ and $b \neq 0$ is a ***pure imaginary number***.

To add a term c to an expression of the form $x^2 + bx$ such that $ax^2 + bx + c$ is a perfect square trinomial is a process called ***completing the square***.

The ***Quadratic Formula*** states that the solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, and c are real numbers and $a \neq 0$.

In the Quadratic Formula, the expression $b^2 - 4ac$ is called the ***discriminant*** of the associated equation $ax^2 + bx + c = 0$.

A system of equations where at least one of the equations is nonlinear is a ***system of nonlinear equations***.

A ***quadratic inequality in two variables*** is an inequality of the form $y < ax^2 + bx + c$, $y > ax^2 + bx + c$, $y \leq ax^2 + bx + c$, or $y \geq ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$

A ***quadratic inequality in one variable*** is an inequality of the form $ax^2 + bx + c < 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$, or $ax^2 + bx + c \geq 0$, where a, b, and c are real numbers and $a \neq 0$

Standards

Common Core:
HSN-CN.A.1, HSN-CN.A.2, HSN-CN.C.7, HSA-CED.A.1, HSA-CED.A.3, HSA-SSE.A.2, HSA-REI.B.4b, HSA-REI.C.7, HSA-REI.D.11, HSF-IF.C.8a

Essential Questions

How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

What are the subsets of the set of complex numbers?

How can you complete the square for a quadratic expression?

How can you derive a general formula for solving a quadratic equation?

How can you solve a nonlinear system of equations?

How can you solve a quadratic inequality?

Games

- Equation Tic-Tac-Toe
- Quadratic Quandary
- Make My Team
- Linear System Sleuths

These are available online in the *Game Closet* at www.bigideasmath.com.

Learning Goals

Solve quadratic equations by graphing.

Solve quadratic equations algebraically.

Define and use the imaginary unit i.

Add, subtract, and multiply complex numbers.

Find complex solutions and zeros.

Solve quadratic equations using square roots.

Solve quadratic equations by completing the square.

Write quadratic functions in vertex form.

Solve quadratic equations using the Quadratic Formula.

Analyze the discriminant to determine the number and type of solutions.

Solve real-life problems.

Solve systems of nonlinear equations.

Graph quadratic inequalities in two variables.

Solve quadratic inequalities in one variable.

Core Concept

Solving Quadratic Equations

By graphing
- Find the x-intercepts of the related function $y = ax^2 + bx + c$.

Using square roots
- Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring
- Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

Core Concept

Zero-Product Property

- If the product of two expressions is zero, then one or both of the expressions equal zero.
- If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

The Square Root of a Negative Number

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $\left(i\sqrt{r}\right)^2 = -r$.

Sums and Differences of Complex Numbers

- To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.
- Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$
- Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

The Quadratic Formula

Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

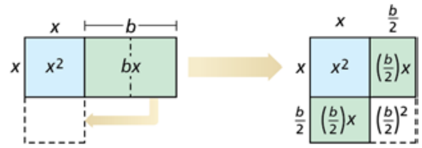
Solving Equations by Graphing

Step 1 To solve the equation $f(x) = g(x)$, write a system of two equations, $y = f(x)$ and $y = g(x)$.

Step 2 Graph the system of equations $y = f(x)$ and $y = g(x)$. The x-value of each solution of the system is a solution of the equation $f(x) = g(x)$.

Completing the Square

- To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.
- In each diagram, the combined area of the shaded regions is $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



- $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for *any* quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for *any* quadratic equation.

Graphing a Quadratic Inequality in Two Variables

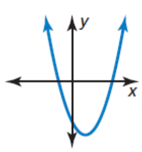
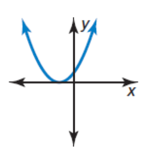
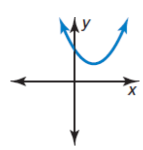
To graph a quadratic inequality in one of the forms above, follow these steps.

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with $\leq$ or $\geq$.

Step 2 Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercepts	 No x-intercepts

Additional Review

- Solving Quadratic Equations Graphically, *p. 94*
- Solving Quadratic Equations Algebraically, *p. 95*
- Solving Quadratic Equations by Completing the Square, *p. 113*
- Writing Quadratic Functions in Vertex Form, *p. 114*
- Solving Equations Using the Quadratic Formula, *p. 122*
- Modeling Launched Objects, *p. 126*
- Solving Systems of Nonlinear Equations, *p. 132*
- Solving Quadratic Inequalities in One Variable, *p. 142*

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 3: Complex Numbers Made Real STEM Video is available online at www.bigideasmath.com.