A **quadratic equation in one variable** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

A **root of an equation** is a solution of the equation.

A **zero of a function** $f$ is an $x$-value for which $f(x) = 0$.

The **imaginary unit** $i$ is the square root of $-1$, denoted $i = \sqrt{-1}$.

A **complex number** is a number written in the form $a + bi$, where $a$ and $b$ are real numbers.

A number written in the form $a + bi$, where $a$ and $b$ are real numbers and $b \neq 0$ is an **imaginary number**.

A number written in the form $a + bi$, where $a = 0$ and $b \neq 0$ is a **pure imaginary number**.

To add a term $c$ to an expression of the form $ax^2 + bx$ such that $ax^2 + bx + c$ is a perfect square trinomial is a process called **completing the square**.

The **Quadratic Formula** states that the solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

In the Quadratic Formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

A system of equations where at least one of the equations is nonlinear is a **system of nonlinear equations**.

A **quadratic inequality in two variables** is an inequality of the form $y < ax^2 + bx + c$, $y > ax^2 + bx + c$, $y \leq ax^2 + bx + c$, or $y \geq ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

A **quadratic inequality in one variable** is an inequality of the form $ax^2 + bx + c < 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$, or $ax^2 + bx + c \geq 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

**Essential Questions**

How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

What are the subsets of the set of complex numbers?

How can you complete the square for a quadratic expression?

How can you derive a general formula for solving a quadratic equation?

How can you solve a nonlinear system of equations?

How can you solve a quadratic inequality?

**Core Concept**

**Solving Quadratic Equations**

**By graphing**

- Find the $x$-intercepts of the related function $y = ax^2 + bx + c$.

**Using square roots**

- Write the equation in the form $u^2 = d$, where $u$ is an algebraic expression, and solve by taking the square root of each side.

**By factoring**

- Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.
**Core Concept**

**Zero-Product Property**
- If the product of two expressions is zero, then one or both of the expressions equal zero.
- If $A$ and $B$ are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

**The Square Root of a Negative Number**
1. If $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.

**Sums and Differences of Complex Numbers**
- To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.
- Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$
- Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

**The Quadratic Formula**
Let $a$, $b$, and $c$ be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Solving Equations by Graphing**

**Step 1**
To solve the equation $f(x) = g(x)$, write a system of two equations, $y = f(x)$ and $y = g(x)$.

**Step 2**
Graph the system of equations $y = f(x)$ and $y = g(x)$. The x-value of each solution of the system is a solution of the equation $f(x) = g(x)$.

**Analyzing the Discriminant of $ax^2 + bx + c = 0$**

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>$b^2 - 4ac &gt; 0$</th>
<th>$b^2 - 4ac = 0$</th>
<th>$b^2 - 4ac &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and type of solutions</td>
<td>Two real solutions</td>
<td>One real solution</td>
<td>Two imaginary solutions</td>
</tr>
<tr>
<td>Graph of $y = ax^2 + bx + c$</td>
<td>Two x-intercepts</td>
<td>One x-intercepts</td>
<td>No x-intercepts</td>
</tr>
</tbody>
</table>

**Completing the Square**
- To complete the square for the expression $x^2 + bx$, add $(\frac{b}{2})^2$.
- In each diagram, the combined area of the shaded regions is $x^2 + bx$.
  Adding $(\frac{b}{2})^2$ completes the square in the second diagram.

- $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

**Methods for Solving Quadratic Equations**

<table>
<thead>
<tr>
<th>Method</th>
<th>When to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>Use when approximate solutions are adequate.</td>
</tr>
<tr>
<td>Using square roots</td>
<td>Use when solving an equation that can be written in the form $u^2 = d$, where $u$ is an algebraic expression.</td>
</tr>
<tr>
<td>Factoring</td>
<td>Use when a quadratic equation can be factored easily.</td>
</tr>
<tr>
<td>Completing the square</td>
<td>Can be used for any quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and $b$ is an even number.</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>Can be used for any quadratic equation.</td>
</tr>
</tbody>
</table>

**Graphing a Quadratic Inequality in Two Variables**
To graph a quadratic inequality in one of the forms above, follow these steps.

**Step 1**
Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola dashed for inequalities with $<$ or $>$ and solid for inequalities with $\leq$ or $\geq$.

**Step 2**
Test a point $(x, y)$ inside the parabola to determine whether the point is a solution of the inequality.

**Step 3**
Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

**What’s the Point?**
The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 3: Complex Numbers Made Real STEM Video is available online at www.bigideasmath.com.

**Additional Review**
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