Big Ideas Math: Algebra 2

Chapter Summary

Chapter 2: Quadratic Functions

Core Vocabulary

A ***quadratic function*** is a function that can be written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

A U-shaped graph of a quadratic function is called a ***parabola***.

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the ***vertex of a parabola***.

The ***vertex form*** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and the vertex is (h, k).

An ***axis of symmetry*** is a line that divides a parabola into mirror images and passes through the vertex.

A quadratic function written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ is in ***standard form***.

The y-coordinate of the vertex of the quadratic function $f(x) = ax^2 + bx + c$, when $a > 0$ is the ***minimum value*** of the function.

The y-coordinate of the vertex of the quadratic function $f(x) = ax^2 + bx + c$, when $a < 0$ is the ***maximum value*** of the function.

A quadratic written in the form $f(x) = a(x - p)(x - q)$, where $a \neq 0$ is in ***intercept form***.

A fixed point in the interior of a parabola, such that the set of all points (x, y) of the parabola are equidistant from the focus and the directrix is the **focus.**

A fixed line perpendicular to the axis of symmetry such that the set of all points (x, y) of the parabola are equidistant from the focus and itself is the ***directrix***.

Games

- Transform Me
- Race for Distance

These are available online in the *Game Closet* at www.bigideasmath.com.

Standards

Common Core:
HSA-CED.A.2, HSA-APR.B.3, HSF-IF.B.4, HSF-IF.B.6, HSF-IF.C.7c, HSF-IF.C.9, HSF-BF.A.1a, HSF-BF.B.3, HSG-GPE.A.2, HSS-ID.B.6a

Learning Goals

Describe transformations of quadratic functions.

Write transformations of quadratic functions.

Explore properties of parabolas.

Find maximum and minimum values of quadratic functions.

Graph quadratic functions using x-intercepts.

Solve real-life problems.

Explore the focus and the directrix of a parabola.

Write equations of parabolas.

Write equations of quadratic functions using vertices, points, and x-intercepts.

Write quadratic equations to model data sets.

Essential Questions

How do the constants a, h, and k affect the graph of the quadratic function $g(x) = a(x - h)^2 + k$?

What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

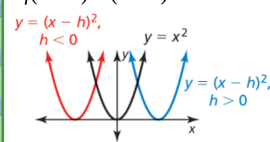
What is the focus of a parabola?

How can you use a quadratic function to model a real-life situation?

Core Concept

Horizontal Translations

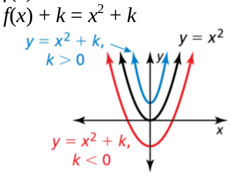
$f(x) = x^2$
$f(x - h) = (x - h)^2$



- shifts left when $h < 0$
- shifts right when $h > 0$

Vertical Translations

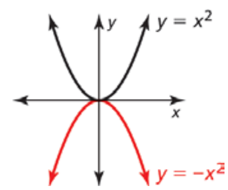
$f(x) = x^2$
$f(x) + k = x^2 + k$



- shifts down when $k < 0$
- shifts up when $k > 0$

Reflections in the x-Axis

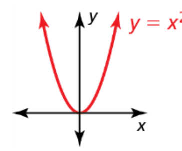
$f(x) = x^2$
$-f(x) = -(x^2) = -x^2$



flips over the x-axis

Reflections in the y-Axis

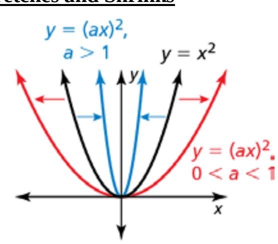
$f(x) = x^2$
$f(-x) = (-x)^2 = x^2$



$y = x^2$ is its own reflection in the y-axis.

Horizontal Stretches and Shrinks

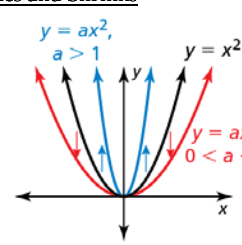
$f(x) = x^2$
$f(ax) = (ax)^2$



- horizontal stretch (away from y-axis) when $0 < a < 1$
- horizontal shrink (toward y-axis) when $a > 1$

Vertical Stretches and Shrinks

$f(x) = x^2$
$a \cdot f(x) = ax^2$

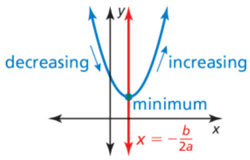


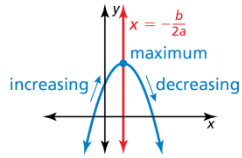
- vertical stretch (away from x-axis) when $a > 1$
- vertical shrink (toward x-axis) when $0 < a < 1$

Core Concept

Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the minimum value of the function when $a > 0$ and the maximum value when $a < 0$.



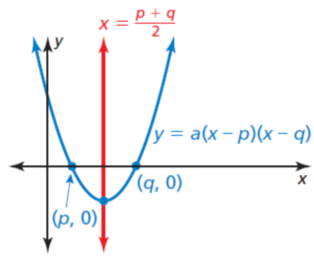


- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

Properties of the Graph of $f(x) = a(x - p)(x - q)$

- Because $f(p) = 0$ and $f(q) = 0$, p and q are the x-intercepts of the graph of the function.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p+q}{2}$.
- The parabola opens up when $a > 0$ and opens down when $a < 0$.



Standard Equations of a Parabola with Vertex at the Origin

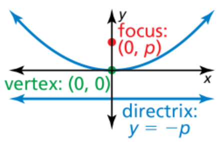
Vertical axis of symmetry ($x = 0$)

Equation: $y = \frac{1}{4p}x^2$

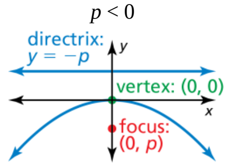
Focus: $(0, p)$

Directrix: $y = -p$

$p > 0$



$p < 0$



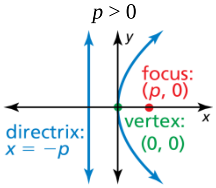
Horizontal axis of symmetry ($y = 0$)

Equation: $x = \frac{1}{4p}y^2$

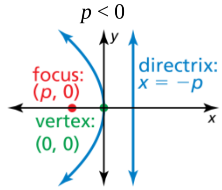
Focus: $(p, 0)$

Directrix: $x = -p$

$p > 0$



$p < 0$



Standard Equations of a Parabola with Vertex at (h, k)

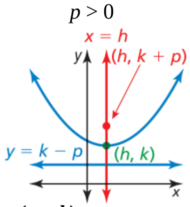
Vertical axis of symmetry ($x = h$)

Equation: $y = \frac{1}{4p}(x - h)^2 + k$

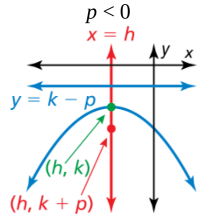
Focus: $(h, k + p)$

Directrix: $y = k - p$

$p > 0$



$p < 0$



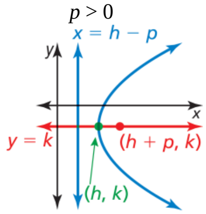
Horizontal axis of symmetry ($y = k$)

Equation: $x = \frac{1}{4p}(y - k)^2 + h$

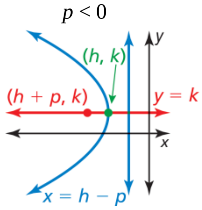
Focus: $(h + p, k)$

Directrix: $x = h - p$

$p > 0$

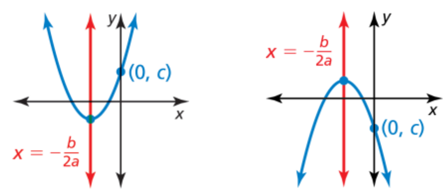


$p < 0$



Properties of the Graph of $f(x) = ax^2 + bx + c$

$y = ax^2 + bx + c,\ a > 0$ $\qquad$ $y = ax^2 + bx + c,\ a < 0$



- The parabola opens up when $a > 0$ and opens down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

Writing Quadratic Equations

Given a point and the vertex (h, k)

- Use vertex form: $y = a(x - h)^2 + k$

Given a point and x-intercepts p and q

- Use intercept form: $y = a(x - p)(x - q)$

Given three points

- Write and solve a system of three equations in three variables.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 2: Solar Energy Using a Parabolic Mirror STEM Video is available online at www.bigideasmath.com.

Additional Review

- Writing Transformations of Quadratic Functions, *p. 50*
- Writing Quadratic Equations to Model Data, *p. 78*