Big Ideas Math: Algebra 2

Chapter Summary

Chapter 1: Linear Functions

Core Vocabulary

The ***parent function*** is the most basic function in a family.

A ***transformation*** changes the size, shape, position, or orientation of a graph.

A ***translation*** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

A ***reflection*** is a transformation that flips a graph over a line called the line of reflection.

A ***vertical stretch*** is a transformation that causes the graph of a function to stretch away from the x-axis when all the y-coordinates are multiplied by a factor a, where $a > 1$.

A ***vertical shrink*** is a transformation that causes the graph of a function to shrink toward the x-axis when all the y-coordinates are multiplied by a factor a, where $0 < a < 1$.

A line that models data in a scatter plot is a ***line of fit***.

The ***line of best fit*** is the line that lies as close as possible to all of the data points in a scatter plot.

The ***correlation coefficient***, denoted by r, is a number from −1 to 1 that measures how well a line fits a set of data pairs (x, y).

A ***linear equation in three variables*** x, y, and z is an equation of the form $ax + by + cz = d$, where a, b, and c are not all zero.

A set of three equations of the form $ax + by + cz = d$, where x, y, and z are variables and a, b, and c are not all zero is a ***system of three linear equations*** in three variables.

A ***solution of a system of three linear equations*** is an ordered triple (x, y, z) whose coordinates make each equation true.

An ***ordered triple*** is a solution of a system of three linear equations represented by (x, y, z)

Standards

Common Core:
HSA-CED.A.2, HSA-CED.A.3, HSA-REI.C.6, HSF-IF.C.9, HSF-BF.A.1a, HSF-BF.B.3, HSF-LE.A.2, HSS-ID.B.6a

Essential Questions

What are the characteristics of some of the basic parent functions?

How do the graphs of $y = f(x) + k$, $y = f(x - h)$, and $y = -f(x)$ compare to the graph of the parent function f ?

How can you use a linear function to model and analyze a real-life situation?

How can you determine the number of solutions of a linear system?

Games

- How Are We Related?
- Transform Me
- Foot and Forearm Examination Activity
- Linear System Sleuths

These are available online in the *Game Closet* at www.bigideasmath.com.

Learning Goals

Identify families of functions.

Describe transformations of parent functions.

Describe combinations of transformations.

Write functions representing translations and reflections.

Write functions representing stretches and shrinks.

Write functions representing combinations of transformations.

Write equations of linear functions using points and slopes.

Find lines of fit and lines of best fit.

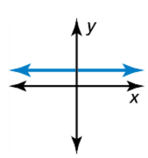
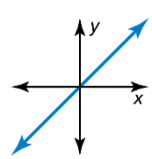
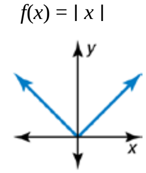
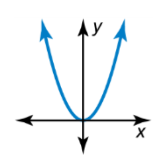
Visualize solutions of systems of linear equations in three variables.

Solve systems of linear equations in three variables algebraically.

Solve real-life problems.

Core Concept

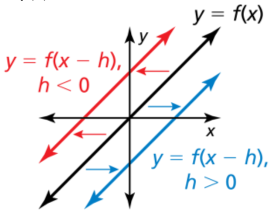
Parent Functions

Family	Constant	Linear	Absolute Value	Quadratic
Rule	$f(x) = 1$	$f(x) = x$	$f(x) = \lvert x \rvert$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

Core Concept

Horizontal Translations

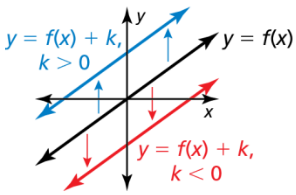
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the inputs before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

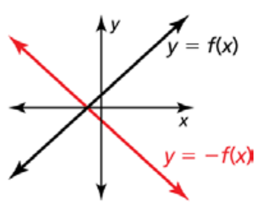
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$. The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



Adding k to the outputs shifts the graph down when $k < 0$ and up when $k > 0$.

Reflections in the x-axis

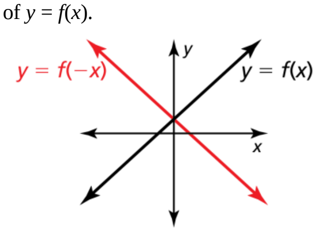
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by -1 changes their signs.

Reflections in the y-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.

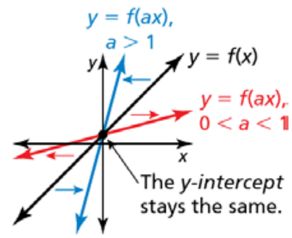


Multiplying the inputs by -1 changes their signs.

Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

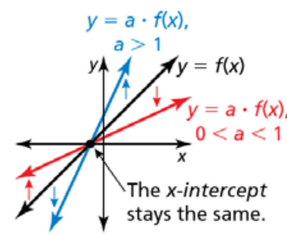
Multiplying the inputs by a before evaluating the function stretches the graph horizontally (away from the y-axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the y-axis) when $a > 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the outputs by a stretches the graph vertically (away from the x-axis) when $a > 1$, and shrinks the graph vertically (toward the x-axis) when $0 < a < 1$.



Finding a Line of Fit

Step 1 Create a scatter plot of the data.

Step 2 Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

Step 3 Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

Step 4 Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

Solving a Three-Variable System

Step 1 Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

Step 2 Solve the new linear system for both of its variables.

Step 3 Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has infinitely many solutions.

Writing an Equation of a Line

Given slope m and y-intercept b

- Use slope-intercept form: $y = mx + b$

Given slope m and a point (x_1, y_1)

- Use point-slope form: $y - y_1 = m(x - x_1)$

Given points (x_1, y_1) and (x_2, y_2)

- First use the slope formula to find m. Then use point-slope form with either given point.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 1: Dirt Bike Trajectory STEM Video is available online at www.bigideasmath.com.

Additional Review

- Describing Transformations, *p. 5*
- Solving Real-Life Problems, *p. 33*